SUR LA QUANTIFICATION DES EFFETS
DU VIEILLISSEMENT DEMOGRAFIQUE :
UNE APPROCHE INTEGREE DE MICRO – MACRO – SIMULATIONS

THESE POUR LE DOCTORAT EN SCIENCES ECONOMIQUES
DE L'UNIVERSITE DE CERGY-PONTOISE

Présentée et soutenue publiquement par

RICCARDO MAGNANI

Le Mercredi, 13 Décembre 2006

Devant le jury composé de :

M. Antoine D'Autume	Professeur, Université de Paris I, *Président*
M. Bruno Decreuse	Professeur, Université de la Méditerranée, *Examinateur*
M. Frédéric Docquier	Professeur, Université Catholique de Louvain, *Rapporteur*
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Introduction Générale

Les pays de l'OCDE vivront dans les décennies futures une phase de changements démographiques importants. L'augmentation de la durée de vie, la réduction du taux de fécondité et le baby-boom qui s'est produit pendant les années 50 et 60, entraînent un vieillissement de la population qui aura des impacts économiques très importants. Le vieillissement de la population provoquera un fort déséquilibre financier des systèmes de retraite par répartition qui représentent le principal pilier de la sécurité sociale en Europe. Les conséquences macroéconomiques seront elles aussi remarquables, du fait que le phénomène démographique aura un impact significatif sur l'offre de travail - à cause de la baisse de la population active - et sur l'offre de capital - à cause des déficits produits par le système de retraite, mais aussi des changements de comportements d'épargne des agents.

L'objectif de cette thèse est de quantifier les impacts économiques du vieillissement démographique, en particulier sur le système macroéconomique et sur le système de retraite, à l'aide d'une approche d'équilibre général.

Dans les 20 dernières années, les *modèles d'équilibre général appliqué* sont devenus, grâce à leurs fondements théoriques, des outils indispensables dans l'évaluation ex-ante des impacts de différentes politiques économiques ou d'autres chocs exogènes. En particulier, les modèles appliqués à générations imbriquées (*overlapping-generations models* ou *OLG models*), représentent un outil approprié dans l'analyse des conséquences du vieillissement démographique. Les modèles de simulation OLG, qui ont été introduits par Auerbach et Kotlikoff (1987) et qui représentent l'extension du modèle théorique à deux générations de Samuelson-Diamond, sont des modèles dynamiques utilisés dans l'analyse des impacts d'un choc démographique sur l'évolution de l'offre du travail et du capital du fait qu'ils permettent de différencier le comportement des générations, en particulier, d'un côté, en termes de participation au marché du travail et, de l'autre, en termes de consommation et d'épargne. Les modèles OLG sont basés sur la théorie du cycle de vie selon laquelle les individus décident du niveau de leur consommation en fonction des flux de revenu perçus tout au long de leur vie. Par conséquent, les individus ont tendance à lisser leur consommation au cours de leur vie et présentent ainsi une propension à épargner élevée pendant la vie active et négative pendant la vie inactive.

A partir du premier modèle OLG appliqué construit par Auerbach et Kotlikoff (1987), la plupart des modèles OLG introduisent un mécanisme de croissance exogène[1], alors que plus rares sont les modèles de croissance endogène.[2]

Un autre élément qui est rarement pris en considération dans les modèles OLG appliqués est la probabilité de survie.[3] La plupart des modèles OLG appliqués, par contre, assument que les individus survivent jusqu'à la dernière période de vie. La probabilité de survie joue bien évidemment un rôle très important dans l'analyse des impacts du vieillissement démographique. D'abord, dans les modèles qui ne prennent pas en considération cet élément, l'évolution démographique est uniquement dictée par le niveau de la fécondité, ce qui implique une difficulté à reproduire d'une façon précise dans le modèle l'évolution démographique prévue par les projections officielles. De plus, une augmentation de la probabilité de survie affecte les décisions de consommation et d'épargne des individus, ce qui provoque des effets non négligeables sur l'accumulation de capital (Yaari, 1965).

Un autre élément récemment introduit dans les modèles OLG appliqués est l'immigration[4], ce qui permet d'évaluer les impacts d'une politique migratoire sur la situation financière du système de retraite.

[1] Voir par exemple, dans le contexte des pays de l'OCDE, Auerbach, Kotlikoff, Hagermann et Nicoletti (1989) ; Hviding et Mérette (1998).

[2] Par exemple, Bouzahzah, de la Croix et Docquier (2002) et Fougère et Mérette (1999) introduisent un mécanisme de croissance lié à l'accumulation du capital humain à la Lucas où, à chaque période, un stock de connaissances est transféré aux plus jeunes et le niveau moyen de connaissances représente une externalité positive pour l'économie.

[3] Voir par exemple Docquier, Liègeois et Stjins (1999) et Storesletten (2000).

[4] Voir par exemple Storesletten (2000) ; Chojnicki, Docquier et Ragot (2005) ; Fougère, Harvey, Mercenier et Mérette (2005).

Si d'un côté les modèles d'équilibre général appliqué sont incontestablement utiles pour évaluer les impacts des politiques économiques, de l'autre, ils font l'objet de critiques parmi lesquelles : 1) Les paramètres utilisés dans le modèle, en général, ne sont pas estimés mais plutôt calibrés sur une année de référence ou bien empruntés de la littérature existante. 2) Les modèles d'équilibre général appliqué utilisent un agent représentatif - ou bien un nombre limité d'agents représentatifs - qui prend ses décisions en maximisant une fonction objective étant donnée une contrainte de ressources. Implicitement, mais sans aucun support théorique, ces modèles se basent sur l'hypothèse que l'agent représentatif représente la collectivité nationale, c'est-à-dire que son comportement optimal agrège parfaitement le comportement optimal de l'ensemble des individus.

En opposition aux modèles d'équilibre général appliqué, à partir des années 80, les *modèles de microsimulation* sont devenus de plus en plus utilisés dans l'analyse des impacts des politiques économiques. L'analyse est effectuée sur un grand échantillon d'agents économiques (individus, ménages, entreprises), ce qui permet de prendre complètement en considération l'hétérogénéité présente dans les micro-données. Les modèles de microsimulation peuvent être utilisés pour quantifier, au niveau individuel, l'impact d'une politique sur le comportement optimal des agents (*behavioural models*) ou, tout simplement, sur leur revenu disponible (*arithmetical models*) sous l'hypothèse que leur comportement reste inchangé.

Les modèles qui considèrent le comportement individuel sont principalement focalisés sur la consommation et sur l'offre de travail. Ces modèles sont basés sur l'estimation du comportement des individus à l'aide d'un modèle économétrique structurel. Ensuite, l'estimation économétrique est utilisée pour simuler l'impact d'une politique économique sur le choix optimal de chaque individu. La microsimulation permet donc d'évaluer les impacts d'une politique au niveau individuel et donc d'analyser les changements dans la distribution des revenus.

Les modèles de microsimulation, aussi, font l'objet d'une critique importante : cette approche ne prend pas en considération les effets d'équilibre général, c'est-à-dire que l'analyse des impacts d'une politique sur le comportement individuel est effectuée sans considérer que le changement de comportement au niveau individuel a un impact au niveau agrégé et, par conséquent, un impact sur le prix d'équilibre (le prix du bien si l'analyse de microsimulation concerne la consommation, ou le salaire si elle concerne l'offre de travail). Par conséquent, le prix utilisé dans la microsimulation pour déterminer le comportement optimal des individus n'est pas correct, ce qui implique une distorsion des résultats.

L'intégration entre les modèles macro d'équilibre général et les modèles de microsimulation, qui permet d'éviter les points faibles de deux méthodologies, apparaît donc comme une étape fondamentale dans l'évaluation des impacts d'une politique et en particulier des effets redistributifs. Cependant, un nombre très limité d'études a utilisé cette approche «micro-macro». Il est possible de distinguer deux types d'approches «micro-macro» : l'approche séquentielle et l'approche intégrée.

L'*approche séquentielle* (voir par exemple Savard (2003)) consiste à itérer le modèle d'équilibre général et le modèle de microsimulation jusqu'à trouver un point fixe qui représente l'équilibre économique. En pratique, la première étape consiste à résoudre le modèle d'équilibre général qui, comme d'habitude, inclut un ou un nombre limité d'agents représentatifs. Ensuite, les prix d'équilibre sont injectés dans le modèle de microsimulation, lequel permet de déterminer l'impact sur les choix individuels, par exemple en termes d'offre de travail. La nouvelle valeur de l'offre de travail sera injectée dans le modèle d'équilibre général, dont la résolution fourni un nouveau vecteur des prix d'équilibre qui seront injectés dans le modèle de microsimulation. Cette procédure itérative est répétée jusqu'à atteindre un point fixe où les prix d'équilibre et, par conséquent, le comportement des individus se stabilisent.

L'*approche intégrée* consiste, par contre, à introduire dans le modèle d'équilibre général tous les individus présents dans une base de données micro. Par exemple, dans Cogneau et Robilliard (2001), le modèle, utilisé pour évaluer les effets distributifs au Madagascar, comprend 4508 individus. Cette approche, par rapport à celle précédente, a sûrement l'avantage de la cohérence du modèle car l'analyse est entièrement effectuée à l'aide d'un seul modèle, alors que l'inconvénient est du type numérique : la présence d'un nombre très élevé d'individus génère un problème de taille du modèle d'équilibre général qui se traduit dans la nécessité d'utiliser des formes fonctionnelles simples et un nombre limité de secteurs et de facteurs productifs.

La thèse, présentée sous forme de 4 papiers, est composée de deux parties. La première partie est consacrée à des analyses de politique économique effectuées à l'aide d'un modèle OLG «standard», c'est-à-dire, d'un modèle d'équilibre général basé sur un agent représentatif. La deuxième partie fournit un apport méthodologique à la littérature existante en proposant une approche intégrée «micro-macro» basée sur la théorie d'agrégation exacte avancée par Anderson, de Palma et Thisse (1992). Cette théorie, qui à notre connaissance n'a jamais été utilisée dans un contexte d'équilibre général, concerne l'agrégation des comportements d'individus hétérogènes faisant des choix discrets.

Dans la suite, nous présentons le contenu des différents chapitres de la thèse.

La **première partie** de la thèse est composée de deux articles concernant l'analyse du problème du vieillissement en Italie, qui représente un cas d'étude emblématique pour deux raisons. D'abord, le phénomène du vieillissement de la population en Italie est, avec le Japon, parmi les plus important au monde. La cause de ce phénomène est principalement liée à la faiblesse du taux de fécondité qui, en 1995, était de 1.19 (Istat, 2004). Actuellement, l'évolution du taux de fécondité montre une hausse (en 2004, il était de 1.33), mais celui-ci reste nettement inférieur à la valeur qui permettrait le renouvellement des générations. Deuxièmement, le cas analysé est intéressant car, à partir des années 90, l'Italie a mis en place plusieurs réformes du système de retraite afin de faire face à une évolution financière du système de retraite qui autrement aurait été catastrophique. En particulier, les réformes introduites en 1992 (réforme Amato) et 1995 (réforme Dini) ont modifié la règle de calcul des pensions en introduisant, d'un côté, une méthode où les pensions sont calculées sur la base des cotisations versées par les travailleurs pendant toute la vie active et, de l'autre, une règle d'indexation des pensions sur la base de l'inflation et non plus sur la base des salaires réels. De plus, en 2004, le gouvernement Berlusconi a introduit une nouvelle réforme qui augmente l'âge minimum de départ à la retraite.

L'objectif de la première partie de la thèse est d'analyser les effets macroéconomiques et les effets sur le système de retraite, à la fois du vieillissement démographique et des réformes récemment introduites.

Le modèle utilisé, comme déjà mentionné, est un modèle OLG appliqué du type Auerbach-Kotlikoff (1987). Le modèle comprend 15 générations (15-24, 25-34, ..., 90-94) et nous supposons que les agents ont des anticipations parfaites et qu'il n'y a pas de contraintes de liquidité.

Les principales différences par rapport au modèle standard Auerbach-Kotlikoff concernent l'introduction d'un mécanisme de croissance endogène à la Lucas (1988), de la mortalité et de l'immigration.

Le mécanisme de croissance endogène est introduit en considérant que les individus âgés de 20-24 ans doivent décider de la fraction du temps à consacrer aux études. Ces individus sont confrontés à un arbitrage entre le travail, qui rapporte un salaire, et l'investissement en éducation, qui permet d'augmenter leurs revenus futurs grâce à une productivité accumulée plus importante. Les jeunes sont supposés effectuer ce choix d'une façon optimale en considérant la valeur actualisée de tous les gains futurs obtenus grâce à la décision de s'éduquer. L'introduction de l'investissement en capital humain apparaît important dans un contexte de vieillissement de la population qui, en augmentant le ratio capital/travail, provoquera une augmentation des salaires et une baisse des taux d'intérêt, ce qui normalement incitera les jeunes à consacrer plus de temps dans l'éducation. De plus, les analyses empiriques (Barro (2001)), montrent la contribution de l'éducation à la croissance. Une augmentation de l'investissement en l'éducation aurait donc un impact positif sur la croissance économique et, par conséquent, sur la situation financière du système de retraite.

L'introduction de la mortalité et de l'immigration nous a permis de reproduire l'évolution démographique Italienne d'une façon très précise. De plus, la distinction entre les personnes nées en Italie et les immigrés crée une hétérogénéité intragénérationnelle, du fait que les immigrés sont caractérisés par un niveau de productivité plus faible que les natifs et par un patrimoine initial faible. L'introduction de l'immigration nous a permit d'évaluer les effets de différentes politiques migratoires, considérées dans le deuxième papier.

Les personnes nées en Italie et les immigrés sont supposés avoir la même structure de préférences. Ils doivent décider du profil intertemporel de leur consommation et du loisir ainsi que la valeur de l'héritage volontaire à laisser à la fin de la dernière période de vie. Par contre, la décision de la fraction du temps à consacrer aux études concerne uniquement les personnes nées en Italie. Les enfants des immigrés, comme supposé dans le modèle de Storesletten (2000), sont considérés identiques aux enfants des natifs.

Le modèle est calibré hors équilibre stationnaire afin de reproduire correctement les données de 1995, en particulier (1) les principales données macroéconomiques (la valeur du PIB, le ratio entre la consommation agrégée et le PIB, le ratio entre les investissements et le PIB, et le ratio entre les dépenses publiques et le PIB), (2) les principaux aspects concernant le système de retraite (le ratio entre le nombre de retraités et le nombre de travailleurs et le ratio entre la dépense agrégée du système de retraite et le PIB, (3) les taux d'occupation des différentes classes d'âge et (4) les propensions à épargner des différentes classes d'âge.

En ce qui concerne le quatrième point, en général, les modèles à générations imbriquées s'intéressent aux profils de consommation, les profils d'épargne étant résiduels. Il en résulte que, conformément à la théorie du cycle de vie, les adultes actifs présenteront une propension à épargner élevée et les personnes âgées une propension à épargner négative. Le vieillissement de la population provoquerait, par conséquent, une forte réduction de l'épargne nationale. Les analyses microéconomiques sur les propensions à épargner des différentes classes d'âge (Poterba (1994)) suggèrent des conclusions complètement différentes, apparemment en contradiction avec la théorie du cycle de vie : les classes les plus âgées présentent des propensions à épargner positives, voir très élevées comme par exemple en Italie et au Japon. Les conséquences macroéconomiques du vieillissement sur l'épargne agrégée seraient dans ce cas sensiblement différentes, et en particulier plus faibles, par rapport à celles mesurées par les modèles précédents.

Afin d'évaluer correctement l'évolution de l'offre de capital, nous avons donc choisi de calibrer les propensions à épargner des classes d'âge dans le but de reproduire les données microéconomiques disponibles. Pour atteindre cet objectif, nous avons calibré le profil intertemporel de consommation (à travers un taux de préférence intertemporel différent pour chaque classe d'âge) et la valeur de l'héritage volontaire laissé à la fin de la dernière période de vie. L'héritage joue en fait un rôle fondamental dans le calibrage de la propension à épargner des classes d'âge. Dans le cas où l'héritage n'est pas pris en compte (comme par exemple dans l'analyse de Miles (1999) sur l'économie anglaise), les personnes âgées, qui sont obligées de consommer la totalité du patrimoine avant la fin de la dernière période de vie, présentent forcément des valeurs des propensions à épargner très négatives et, par conséquent, qui ne sont pas cohérentes avec les données réelles.

Le modèle est utilisé dans les deux premiers papiers pour simuler les impacts de différents chocs sur l'économie Italienne. Le **premier papier**, qui présente la description détaillée du modèle, analyse les effets sur le système macroéconomique Italien, et en particulier sur le système de retraite, de l'introduction des réformes Amato et Dini qui prévoient (1) l'indexation des pensions à l'inflation et (2) une méthode de calcul des retraites sur la base des cotisations versées pendant toute la durée de travail. L'objectif de ce papier est d'analyser l'efficacité de ces réformes. Pour cette raison nous avons comparé les résultats des simulations qui considèrent l'introduction des réformes Amato et Dini avec ceux obtenus en supposant que ces réformes n'avaient pas été introduites. Les résultats de cette comparaison montrent que les réformes étaient absolument indispensables dans le but de réduire les déficits du système de retraite qui autrement auraient été énormes. Un autre aspect positif des réformes introduites dans les années 90 est lié aux impacts sur la situation macroéconomique : la réduction des déficits futurs du système de retraite permettra de réduire l'effet négatif sur l'épargne agrégée et, par conséquent, de produire une évolution plus favorable du PIB et du PIB par tête. Cependant, il est aussi possible d'affirmer que ces réformes ne sont pas suffisantes, ni à court terme, ni à long terme : dans la période 2025 - 2045, des déficits de l'ordre de 3-5 % par rapport au PIB sont prévus et, à long terme, l'équilibre du système de retraite ne sera pas atteint.

La mise en place d'une nouvelle réforme était donc indispensable. Pour cela, en 2004, le gouvernement Berlusconi a introduit une nouvelle réforme qui prévoit l'augmentation de l'âge minimum de départ à la retraite. Alors qu'avec les réformes Amato et Dini chaque travailleur pouvait décider de partir à la retraite entre 57 et 64 ans, avec la nouvelle réforme l'âge minimum de départ à la retraite est fixé à 60 ans à partir de 2008 et à 61 à partir de 2010. De plus, en 2015, le gouvernement devra décider si augmenter ultérieurement l'âge de départ à la retraite et le fixer à 63 ans.

Le **deuxième papier** présente l'analyse des impacts de la réforme Berlusconi et d'autres réformes qui pourraient être introduites dans le but d'atteindre l'équilibre du système de retraite à long terme. Nous montrons que les impacts en termes d'évolution du déficit du système de retraite par rapport au PIB sont très positifs à court terme et à moyen terme, alors que, à partir de 2040, cette réforme devient complètement inefficace. Une analyse de comptabilité générationnelle (Auerbach, Gokhale and Kotlikoff (1994)) montre aussi la perte subie par les générations obligées à travailler davantage.

Etant donné que l'augmentation de l'âge de la retraite prévue par le gouvernement Berlusconi ne sera pas suffisante à rétablir l'équilibre du système de retraite à long terme, nous avons considéré d'autres réformes, complémentaires à la réforme Berlusconi, qui pourraient résoudre ce problème : des politiques migratoires et la réduction de la valeur de la pension.

La **deuxième partie** de la thèse traite de la méthodologie d'intégration entre les modèles macro d'équilibre général et les modèles de microsimulation. Comme déjà mentionné, les modèles d'équilibre général se basent sur un agent représentatif qui implicitement, mais sans aucun support théorique, devrait agréger les comportements des individus.

Dans cette partie de la thèse nous utilisons le résultat théorique d'agrégation présenté par Anderson, de Palma et Thisse (1992). Cette théorie d'agrégation montre que des individus qui doivent choisir une quantité continue parmi un ensemble d'alternatives discrètes peuvent être agrégés en un individu représentatif ayant des préférences CES. Ce résultat d'agrégation fournit un lien naturel entre les modèles de microsimulation, où un ensemble d'individus hétérogènes font des choix discrets, et les modèles d'équilibre général.

Il s'agit donc d'une approche «micro-macro» intégrée qui, par rapport aux modèles existants, ne nécessite pas d'introduire dans le modèle macro la totalité des individus présents dans la base de données micro. En fait, dans le modèle macro, il est suffisant d'introduire un (ou plusieurs) individu représentatif dont le comportement agrège parfaitement le comportement des individus. Etant donné le résultat d'agrégation, l'équilibre obtenu dans le modèle d'équilibre général est correct et peut être injecté dans le modèle de microsimulation pour effectuer une analyse au niveau individuel, en particulier pour déterminer les changements du comportement et du revenu de chaque individu et, par conséquent, l'impact sur la distribution des revenus et sur les inégalités.

L'apport à la littérature existante est donc double. Premièrement, les paramètres des fonctions qui décrivent le comportement de l'agent représentatif dans le modèle d'équilibre général ne sont pas calibrés mais estimés sur des micro-données. De plus, cette approche «micro-macro» permet d'effectuer une analyse au niveau individuel.

Le **troisième papier** illustre l'utilité de cette méthodologie dans un contexte de vieillissement de la population d'une économie fictive mais réaliste. Nous utilisons un modèle OLG simple et une base de données micro générée par ordinateur afin d'évaluer les impacts potentiels du vieillissement de la population sur l'évolution de la distribution des revenus et des inégalités. Dans cette économie fictive, les individus sont regroupés sur la base des caractéristiques démographiques (âge et sexe) et doivent choisir entre travailler et consommer du loisir et, dans le cas où ils travaillent, dans quelle profession. Nous considérons cinq classes d'âge qui travaillent (15-24, ..., 55-64). Par conséquent, il y a 10 groupes d'individus et 10 agents représentatifs dans le modèle OLG. Pour chaque groupe, nous déterminons une fonction d'offre de travail relative à chaque profession qui agrège parfaitement les choix des individus et qui est ensuite introduite dans le modèle OLG.

Le modèle OLG est calibré à l'équilibre stationnaire. Nous introduisons un choc démographique représenté par une baisse des taux de fécondité qui provoque, au niveau macro, une augmentation des salaire et une réduction du taux d'intérêt. Ensuite, les prix d'équilibre sont injectés dans le modèle de microsimulation pour déterminer les changements de comportement au niveau individuel. Nous montrons d'abord que les propriétés d'agrégation sont respectées, c'est-à-dire que l'agrégation du comportement des individus en terme d'offre de travail dans chaque profession coïncide presque parfaitement avec l'offre de travail déterminée dans le modèle macro.

Le modèle de microsimulation est utilisé pour déterminer, pour chaque individu, le nouveau comportement en termes d'offre de travail et, par conséquent, le nouveau revenu perçu. Le papier se conclu avec une analyse sur la distribution des revenus et sur les inégalités dans cette économie fictive.

Dans le **quatrième papier**, nous appliquons cette méthodologie à l'économie Canadienne dans le but d'évaluer les impacts du vieillissement démographique sur le système macroéconomique et, en particulier, sur la distribution des revenus. Dans cette analyse nous avons utilisé la base de données FMGD (Fichier de micro-données à grande diffusion) de l'année 2001 comprenant 62077 individus. Chaque individu doit choisir entre travail et loisir et, dans le cas où il décide de travailler, dans quelle profession. De plus, les individus appartenant à la première classe d'âge (15-24) doivent décider du niveau d'éducation. En particulier, nous considérons 10 types de professions et 5 niveaux d'études. Les individus sont regroupés sur la base des caractéristiques démographiques (âge, sexe et niveau d'éducation). Par conséquent, le modèle OLG comprend 50 agents représentatifs qui agrègent parfaitement le comportement individuel en terme d'offre de travail par profession et de choix de type de diplôme.

Ensuite, nous introduisons dans le modèle OLG le choc démographique représenté par l'évolution des taux de fécondité et de mortalité que nous avons calibré dans le but de reproduire l'évolution démographique Canadienne. Le modèle OLG permet de déterminer l'évolution des salaires d'équilibre pour chaque profession et celle du taux d'intérêt. Ensuite, ces résultats sont injectés dans le modèle de microsimulation, ce qui nous permet d'analyser l'impact du vieillissement démographique sur la distribution des revenus et sur le niveau d'inégalité mesuré, pour différents types d'individus, en utilisant les indices de Gini.

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Population ageing in Italy and efficiency of the Amato and Dini reforms: An overlapping-generations general equilibrium model

Abstract

In this paper we analyse the effect of the population ageing in Italy on the economic system and, in particular, on the pension system in order to evaluate the efficiency of the reforms introduced during the Nineties (Amato reform in 1992 and Dini reform in 1995). An overlapping-generations general equilibrium model is built in order to evaluate both the macroeconomic impacts and the evolution of the pension system in terms of financial deficit and intergeneration equity.

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1 Introduction

Industrialised countries will face in the next decades a period of important demographic changes. The increase in the life expectancy, the reduction in fertility rates, and most of all the baby-boom produced during the Fifties and Sixties have induced a population ageing which will put the financing of the social security systems under considerable stress.

To face this problem, most European countries have recently introduced pension system reforms. Even if European pension systems remain essentially different, some similar rules have been introduced in order to reduce the pension expenditure burden: the indexation of pension benefits to prices, the increase in the retirement age and the increase of the role of private funding. However, the Pay-As-You-Go system is still largely the most important pillar of European pension systems. Italy is no exception, and provides an interesting case because it was among the first countries to handle this problem.

Moreover, Italy is an important case study because the population ageing phenomenon is one of the most important in the world. The demographic projections based on the central hypothesis presented by Istat[1] show that:

- The number of people aged 65 and more will increase from 18% in 2000 to 32.3% in 2050.
- Simultaneously, the number of young people aged less than 19 will go down from 19.8% in 2000 to 15.6% in 2050, and the number of people aged 20-34 will decrease from 22.5% in 2000 to 14% in 2050.
- The active population - the number of people between 15 and 64 years old - will drop by 30% from year 2000 to 2050.
- The old-age dependency ratio - the ratio of the number of people aged 65 or more to the active population - will increase from 26.6% in 2000 to 63.5% in 2050.
- The total population will decrease, from 2015 onwards. That is mostly due to a reduction of the fertility rate (which is already lower than the value which allows the renewal of the generations) not sufficiently compensated by the migratory flows.

[1] Istat (2001), Previsioni della popolazione residente per sesso, età e regione. Base 1.1.2001.

Figures 1, 2 and 3 in Appendix show respectively, over the period 1950-2050, the evolutions of the population structure, the old-age dependency ratio and the total active population.

This demographic trend induces significant problems related to the current system of financing the pensions. The increase in the old-age dependency ratio causes a strong discrepancy between the value of the social security contributions perceived and the pension benefits paid. This structural deficit threatens the respect of the criteria of the European Stability Pact.

During the Nineties, two pension system reforms were introduced in order to solve this problem: the Amato reform in 1992 and Dini reform in 1995. However, even if these reforms predict a strong reduction in the value of the future pension benefits, they will allow controlling only for the evolution of the ratio between the total expenditure of the pension system and the GDP, but do not permit to reach the financial equilibrium of the pension system.

Many papers in the literature studied the problem of the population ageing, particularly in the Italian context. For example, Brugiavini and Fornero (1999), Castellino and Fornero (1997), Ferraresi and Fornero (2000), Franco (2000), use a partial equilibrium approach to analyse the impact on the Italian pension system, under the assumption that wages and GDP grow at a constant rate and that interest rates remain constant over time. However, ageing constitutes a major change in the macroeconomic environment whose repercussions will not only affect the social security system. It is clear that the evolution of the labour and capital supply will considerably affect the level of the main macroeconomic variables (the aggregate consumption, the aggregate saving, and the national production) and, consequently, the level of the national welfare. In this study, we will use a general equilibrium approach that makes possible the evaluation of the effects of population ageing and those related to the introduction of pension reforms on the economy as a whole, and particularly, on the labour market (thus, on wages which directly influence the evolution of the social security contributions) and the capital (thus, on the investments and on the interest rates which influence the GDP growth). Several studies used an overlapping-generations (OLG) general equilibrium model in order to evaluate the impacts of population ageing on developed countries, see Auerbach *et al.* (1989), Hviding and Mérette (1998), Fougère and Mérette (1999), Miles (1999), Bouzahzah, de La Croix and Docquier (2002).

In this paper, we specifically investigate the Italian case. The innovative part of our study is in the introduction in our OLG model of the mortality, the immigration and the accumulation of the human capital. The introduction of mortality and immigration enables us to reproduce the demographic evolution in a very precise way. In particular, the distinction between the individuals who are born in Italy and the immigrants is a source of heterogeneity. This heterogeneity comes from the realistic assumption that the immigrants are characterised by a productivity level lower than the natives and by a weak initial capital (nearly equal to zero). The introduction of human capital enables us to introduce an endogenous growth mechanism based on the average level of knowledge present in the economy. Human capital is modelled by considering that people belonging to the first age group must decide the level of investment in education. That seems important since the decisions to invest or not in human capital will depend on the relative factor prices which are expected to considerably fluctuate because of ageing.

Furthermore, our approach differs from the previous analysis by the calibration technique we applied: the model is calibrated "out of steady state" in order to correctly reproduce the data of the year 1995.

The aim of this paper is to analyse the impacts of ageing on the Italian economy and, in particular, on the pension system in order to evaluate the efficiency of the reforms introduced during the Nineties. We thus try to answer the following question: are the Amato and Dini reforms sufficient, or is it necessary to introduce some new reforms?

The remainder of the paper is organised as follows. In the following section, we describe the characteristics of the Italian pension system and the reforms introduced in the Nineties. In Section 3, we describe the method applied in order to reproduce in the model the Italian demographic evolution. We analyse, in Section 4, the structure of the model and its "out of steady state" calibration. Sections 6 and 7 present the results of the simulations concerning the population ageing and the introduction of the Amato and Dini reforms. The following section presents two sensitivity analyses. The last section concludes.

2 The Italian pension system and the reforms of the Nineties

The Italian pension system is almost entirely composed of a compulsory public system that is financed as a Pay-As-You-Go system. The expenditures related to the pension system in Italy have strongly increased since the Sixties.[2] An important anomaly of the Italian pension system is that there is not a clear separation between the pension system in the strict sense and a system of social aid, in which benefits are not related to contributions.

In particular, the Italian pension system includes:

- Pensions related to activity, old-age pensions, disability pensions, pensions paid in case of occupational disease and industrial accidents.
- Reversibility pensions, paid to the survivors of retirees.
- Pensions paid to old people aged 65 and more with an insufficient income.

Moreover, until 1992, the Italian pension system was characterised by a very large number of funds and schemes, in which contributions and benefit rules varied according to sectors (private or public sector, or self employment). In particular, the existence of two determinants of the retirement decision poses a problem. On the one hand, the possibility to retire at a minimum age (*pensione di vecchiaia*) and on the other hand, the possibility of an early retirement with a minimum year number of contributions (*pensione di anzianità*).

At the beginning of the Nineties, the Italian pension system was characterised by three serious problems:[3]

- A very high financial deficit.
- An important distributive inequity: the existence of different pension plans benefits more to self-employed workers with respect to the employed.[4] Public administration employees are also advantaged to the employees of private companies (before the

[2] They accounted for 5% of the GDP in 1960, 10.2% in 1980 and 15.7% in 1999 (Franco (2000)).

[3] Castellino and Fornero (1999).

[4] Peracchi and Rossi (1995) estimate that the implicit rate of return on contributions obtained by self-employed workers was twice higher than the implicit rate of return on contributions obtained by the employees.

reform, for the primers, the pension was calculated on the basis of wages perceived the last year of work compared with the 5 last years for the latters).

- Incentives to early retirement: the absence of a link between the value of the pension benefits and the retirement age, pushes individuals to be leave his job as soon as possible, as if there was an implicit tax, very high, on the prolongation of the activity.[5] That can explain the weakness of the activity participation of men and women aged 55-65.

Our objective is to evaluate the evolution of the Italian pension system. As we have already mentioned, during the Nineties two reforms were introduced in order to reduce total expenditures on pensions, to harmonise the different pension regimes, and to increase the fairness of the pension system: the Amato reform (1992) and the Dini reform (1995). The two reforms predict a time-scale of their application: the Dini reform introduced a new rule for people who entered the labour market since 1995 and for people having less than 18 years of contributions in 1995.

The Italian pension system is thus "in transition" and an evaluation of its future evolution requires the consideration of the effects of these reforms since their implementation. These reforms will thus constitute our political economy shocks. We now describe the principal elements of these reforms.

2.1 The Amato reform (1992)

The principal innovations of the Amato reform are as follows:

- To obtain the pension related to the retirement age (*pensione di vecchiaia*), the retirement age increased from 55 to 60 years for the women and from 60 to 65 years for the men, and the minimum number of years of contributions passes from 15 to 20 years.

- To obtain the pension related to the number of years of contributions paid (*pensione di anzianità*), the number of years of contributions increases gradually in the public sector to 35 years, as applied in the private sector.

[5] Brugiavini (1997).

- The introduction of regulated pension funds.

- The indexation of pension benefits on inflation, not on real wages.

2.2 The Dini reform (1995)

The Dini reform determines the transition from a system in which pension benefits are computed on the basis of past salaries to a system in which they are computed on the basis of the contributions paid during the whole working life. With this reform, the retirement is allowed for people aged 57 or more with at least 5 years of contributions, or for workers who have 40 years of contributions.

Workers can decide to retire between 57 and 65, and pensions related to the number of years of contributions (*pensione di anzianità*) will disappear. The goal of this reform is thus to penalise early retirement.

Pension benefits are computed as follows:

- For people who start working after 1995, pension benefits are computed with the *contribution based method*: the contributions paid during the whole working life are virtually capitalised on the basis of the GDP growth rate. The capitalised value of contributions is multiplied by a transformation coefficient fixed by law and depending on the retirement age of individuals.[6]

- For people who in 1995 had more than 18 years of contributions, pension benefits are computed with the *earning based method*, i.e. on the basis of the average salary earned in the last 10 years of working.

- For people who in 1995 had less than 18 years of contributions, pension benefits are computed with the *pro-rata method*: pension benefits are given by a weighted average between the pensions computed with the contribution and pensions computed with the earning based method.

[6] The transformation coefficients are reported in Table 30 in Appendix.

3 Reproduction of the demographic evolution

The model presented in this paper is an overlapping-generations model in which 15 age groups, indicated by $g(k)$ with $1 \leq k \leq 15$, coexist at each period t:

g(1)	20 - 24
g(2)	25 - 29
g(3)	30 - 34
g(4)	35 - 39
g(5)	40 - 44
g(6)	45 - 49
g(7)	50 - 54
g(8)	55 - 59
g(9)	60 - 64
g(10)	65 - 69
g(11)	70 - 74
g(12)	75 - 79
g(13)	80 - 84
g(14)	85 - 89
g(15)	> 90

We suppose that immigration concerns only the age group 30-34, then we have to distinguish - for the following age groups - two types of individuals, indicated by z: people born in Italy (it) and immigrants (im). At the end of each period, people belonging to the last age group ($k = 15$) die, a fraction of people belonging to the other classes dies, and a new generation enters the active population.

The demographic evolution is described by the following equations:

$$POP^{it}_{g(1),t+4} = \sum_{k=1}^{4} \eta_{g(k),t} \cdot \left(POP^{it}_{g(k),t} + POP^{im}_{g(k),t} \right) \quad (1)$$

$$POP^{im}_{g(3),t} = imm_{g(3),t} \quad (2)$$

$$POP^{z}_{g(k+1),t+1} = POP^{z}_{g(k),t} \cdot \gamma_{g(k+1),t+1} \quad (3)$$

Equation (1) indicates that the number of people born in t and belonging to the first age group (20-24) in t+4 ($POP^{it}_{g(1)t+4}$) depends on the number of individuals (born in Italy or immigrants) between 20 and 39 years old and on the parameters $\eta_{g(k),t}$ (with $k = 1, ..., 4$) representing the fertility rates which are differentiated by the age of the parents.

Equation (2) indicates that the number of immigrants belonging to the age group 30-34 ($POP^{im}_{g(3),t}$) is given by the immigration flows ($imm_{g(3),t}$).

Equation (3) indicates that the number of people - born in Italy and immigrants - belonging to the the age group $g(k+1)$ in $t+1$ depends on the number of people belonging to the age group $g(k)$ in t and on the survival probability $\gamma_{g(k+1),t+1}$. We used the same survival probabilities and the same fertility rates for people born in Italy and immigrants.[7]

Given that we only consider people aged 20 and more, our purpose is to reproduce the demographic evolution of the population aged 20 and more, and in particular:

- The dependency ratio, i.e. the ratio of people aged 65 and more to people aged 20-64.
- The structure of the population, i.e. the ratio of the number of people belonging to a given age group to the total population.
- The total population.

As we early noted, immigrants come in Italy when aged between 30 and 34. We adopt migratory flows between 100,000 and 120,000 individuals per year since 1990, following Istat's assumptions. Return migration is not considerate in the model. Concerning the first 9 age groups (20-64), we used the survival rates presented by Istat (2000). We calibrate the survival probabilities for the other age groups ($\gamma_{g(k),t}$ with $k \geq 10$) and the fertility rates ($\eta_{g(k),t}$ with $k \leq 4$) in order to reproduce the Italian demographic evolution. In particular, the fertility rates are differentiated by the age of the parents and are calibrated by respecting the proportions between the fertility rates present in data.[8]

In the Appendix, we report the survival probabilities presented by Istat for 2000 (Table 4), the (calibrated) survival probabilities used in the model for people over 65 (Table 5), and the (calibrated) fertility rates used in the model (Table 6).

The results of the reproduction of the Italian demographic evolution in the model are represented in the Appendix: Figure 7 concerns the dependency ratio, Figure 8 concerns

[7] Mayer and Riphahn (1999) estimated that fertility rates tend to converge between immigrants and natives.

[8] In 1994, for 1000 women, the fertility rate was 40.4 for the age group 20-24, 84.7 for the age group 25-29, 74.6 for the age group 30-34, and 31.1 for the age group 35-39 (Istat, 1999).

the population structure by age, and Figure 9 concerns the total population. In particular, we can see that the dependency ratio, which represents the most important demographic variable, is nearly perfectly reproduced.

4 The model

4.1 The generations

4.1.1 Characteristics of the model

The OLG model includes 15 age groups. We suppose that there exists a representative agent for people born in Italy and a representative agent for immigrants (intra-generational heterogeneity), that agents have perfect foresight and that there are no liquidity constraints. Each period is given by 5 years.

People aged less than 20 are not considered, since they are supposed completely dependent of their family.

As we have already mentioned, at the end of each period, people belonging to the last age group die, a fraction of people belonging to the other classes dies on the basis of the mortality rates, and a new generation enters the active population.

People who die in the last period (95 years old) decide to leave a bequest to the other generations, on the basis of a maximisation process of their utility function: in this case, there are voluntary bequests. On the other hand, people belonging to the other age groups, in the case of premature death, do not program the value of their final wealth: in this case, there are involuntary bequests. Voluntary and involuntary bequests are uniformly distributed among the other generations.

In this model, people belonging to the first 9 age groups supply labour. Labour supply is endogenous for the first 7 age groups. In particular, people belonging to the first age group (20-24 years old) must decide the fraction of time to devote to the human capital formation. The following age groups, until the class 50-54, must decide the fraction of time to devote to work and to leisure. With regard to the two last age groups who work (55-59 and 60-64 years old), the fraction of people which works is exogenously fixed, according to the 1995 data.

Immigrants and people born in Italy have the same structure of preferences. They must decide the intertemporal profile of consumption and leisure as well as the value of the voluntary bequest that will be left at the end of the last period of life. On the other hand, the decision about the fraction of time to devote to study concerns only people born in Italy. Moreover, immigrants differ from people born in Italy by a lower level of productivity and we assume that immigrants enter in Italy with no capital. The children of immigrants are considered identical to the children of people born in Italy. Consequently, they must decide the fraction of time to devote to studying and the difference in productivity disappears.

4.1.2 Human capital

Our model is an endogenous growth model based on human capital accumulation. We introduce a human capital accumulation *à la* Lucas (1988). Other OLG models that include an endogenous growth mechanism are provided by Fougère and Mérette (1999) and Bouzahzah et al. (2002). In those models economic growth is given by the transmission of human capital to the new generations, so individuals belonging to the first age group receive a human capital stock as bequest. In this paper, economic growth is related to the average human capital stock that represents the average level of knowledge in the economy. We assume that this knowledge level affects in the same way the productivity for all the individuals that coexist at the same period.

In particular, the individual productivity level depends on three elements:

i) Productivity related to his age, and thus on his experience, measured by $EP_{g(k)}$. This component exerts a quadratic form:

$$EP_{g(k)} = \theta + \theta_1 k + \theta_2 k^2 \tag{4}$$

with $1 \leq k \leq 9$, because only the first 9 age groups work.

ii) Productivity related to education, measured by $HC_{g(k),t}$, which is a concave function of time spent in school:

$$HC_{g(1),t} = \left[\Delta \cdot \left(1 - l^{it}_{g(1),t}\right)\right]^{\alpha_{HC}} \tag{5}$$

Here, $l^{it}_{g(1),t}$ is the fraction of time that a representative individual of the age group 20-24 spends working.[9] The stock of human capital accumulated by the individuals that belong to the first age group depends on the number of years devoted to studying $\left[\Delta \cdot \left(1 - l^{it}_{g(1),t}\right)\right]$. In the next periods, human capital accumulated by an individual depreciates, according to a rate δ_{HC}:

$$HC_{g(k),t+k-1} = HC_{g(1),t} \cdot [1 - (k-1) \cdot \delta_{HC}] \tag{6}$$

with $k > 1$.

iii) Productivity related to the average level of knowledge in the economy, H_t; the average level of knowledge is measured by the weighted average of the stocks of human capital accumulated by each class that coexists at the same period:

$$\overline{H}_t = \frac{\sum_k HC_{g(k),t} \cdot l^{it}_{g(k),t} \cdot POP^{it}_{g(k),t}}{\sum_k l^{it}_{g(k),t} \cdot POP^{it}_{g(k),t}}$$

where $l^{it}_{g(k),t}$ indicates the fraction people born in Italy belonging to the age group $g(k)$ that work. The productivity growth rate (g_{H_t}), that represents the steady state growth rate of the variables in per capita terms, is endogenous and related to the average level of knowledge:

$$g_{H_t} = \frac{H_{t+1} - H_t}{H_t} = \chi \cdot \overline{H}_t^{\frac{1}{\alpha_{HC}}} \tag{7}$$

where χ is a constant parameter.[10] As no individual can influence, by his decision to study, the value of this index, this stands as a positive externality.

An individual's total productivity ($A^z_{g(k),t}$) is given by the product of the previous three elements:

$$A^z_{g(k),t} = EP_{g(k)} \cdot HC_{g(k),t} \cdot H_t \cdot \theta^z \tag{8}$$

[9] An equivalent interpretation of $l^{it}_{g(1),t}$ is the fraction of young people born in Italy belonging to the age group 20-24 that works.

[10] Note that $\overline{H}_t^{\frac{1}{\alpha_{HC}}}$ represents a weighted average of the number of years devoted to studying by individuals that coexist in t, i.e. $\overline{H}_t^{\frac{1}{\alpha_{HC}}} = \left\{\frac{\sum_k \left[\Delta \cdot \left(1 - l^{it}_{g(1),t-k+1}\right)\right]^{\alpha_{HC}} \cdot l^{it}_{g(k),t} \cdot POP^{it}_{g(k),t}}{\sum_k l^{it}_{g(k),t} \cdot POP^{it}_{g(k),t}}\right\}^{\frac{1}{\alpha_{HC}}}$. In particular, at the steady state where the education level is the same for each individual, we have that $\overline{H}_t^{\frac{1}{\alpha_{HC}}} = \Delta \cdot \left(1 - l^{it}_{g(1)}\right)$, so the productivity growth rate is proportional to the number of years devoted to studying, $g_h = \chi \cdot \Delta \cdot \left(1 - l^{it}_{g(1)}\right)$, as in Lucas (1988).

with θ^{it}=1 and θ^{im}=0.87, because the total productivity of the immigrants is supposed to be lower by 13%.[11]

4.1.3 The intertemporal utility function

People born in Italy and immigrants have utility functions of similar form. The intertemporal utility function is given by three elements:

i) The present value of the sum of utilities of future consumptions, weighted by the survival probability.

ii) The present value of the sum of utilities of future leisure, weighted by the survival probability.

iii) The present value of the utility of the bequest left at the end of the last period of life, weighted by the survival probability.

The expected lifetime utility for the generation born in t is the following:[12]

$$\begin{aligned} U_t^z \quad = \quad & \sum_k \log\left(c^z_{g(k),t+k-1}\right)\cdot B_{g(k)}\cdot\Omega_{g(k),t+k-1} \\ & +\sum_k \epsilon_{g(k),t+k-1}\cdot\log\left[\Delta\cdot\left(1-l^z_{g(k),t+k-1}\right)\right]\cdot B_{g(k)}\cdot\Omega_{g(k),t+k-1} \\ & +\beta_{BEQ}\cdot\log\left(BEQ^z_{t+14}\right)\cdot B_{g(15)}\cdot\Omega_{g(15),t+14} \end{aligned} \tag{9}$$

with $1\leq k\leq 15$ for people born in Italy and $3\leq k\leq 15$ for immigrants.

The following notations have been used: Δ stands for the number of years that constitute one period (5 years), $c^z_{g(k),t}$ is consumption of the age group $g(k)$ for one period,

[11] Storesletten (2000) estimates for the United States that the productivity of people who immigrate at 37 years old is lower by 13% with respect to the natives. This assumption implies that immigrants have a level of productivity related to education lower by 13% compared to natives. In fact, we can suppose that an immigrant and a native, with the same age, have the same productivity related to the experience (EP) and that they profit in the same way of the knowledge present in the economy (H). By considering equation (5), this assumption implies that immigrants have a stock of human capital lower by 10% compared to natives.

[12] We use a logarithmic utility function since the unique value for the intertemporal substitution compatible with a balanced growth path is equal to 1. This implies that growth does not affect the quality of leisure.

$B_{g(k)}$ is the actualisation factor, with $B_{g(k)} = \prod_{s=1}^{k} \frac{1}{1+\rho_{g(s)}}$ where $\rho_{g(k)}$ is the intertemporal preference rate of an individual belonging to the class $g(k)$, $\epsilon_{g(k),t}$ measures the intensity of the preference for leisure with respect to consumption and β_{BEQ} is the intensity of the preference for bequests.

$1 - l^z_{g(k),t}$ with $k > 1$ represents the fraction of time that the class $g(k)$ devotes to leisure, whereas for the first age group ($k = 1$), it represents the fraction of time devoted to studying.

$\Omega_{g(k),t}$ is the probability that a person that belongs to the age group $g(k)$ is alive in t. Clearly, $\Omega_{g(1),t} = 1$ and:

$$\Omega_{g(k),t} = \prod_{w=1}^{k} \gamma_{g(w),t-k+w} \qquad (10)$$

where $\gamma_{g(k),t}$ is the probability that an individual belonging to the age group $g(k-1)$ in *t-1* survives at the end of the period.

4.1.4 Intertemporal budget constraint

Concerning the intertemporal budget constraint we have to consider two cases:

i) The budget constraint for people who live until the last age group (95 years old). In this case the final wealth is voluntary left as a bequest. The present value of the final wealth is given by the difference between the present value of future incomes and the present value of future consumption. Then, the present value of the final wealth will be equal to the present value of bequest:

$$\sum_{k=1}^{15} R_{t+k-1} \cdot \left(m^z_{g(k),t+k-1} - d^z_{g(k),t+k-1} \right) = R_{t+14} \cdot p_{t+14} \cdot BEQ^z_{t+14} \qquad (11)$$

R_t is the actualisation factor with $R_{t+k-1} = \prod_{s=t+1}^{t+k-1} \left(\frac{1}{1+r_{net_s}} \right)$, where r_{net_t} is the net interest rate.

The total net income, indicated by $m^z_{g(k),t}$, is given by net labour incomes, net pension benefits and inheritances :

$$\begin{aligned} m^z_{g(k),t} \;=\; & (1 - \tau_t - \tau_{cs}) \cdot l^z_{g(k),t} \cdot w_{lab,t} \cdot A^z_{g(k),t} \\ & + (1 - \tau_t) \cdot PP^z_{g(k),t} \cdot \Delta \cdot Pens^z_{g(k),t} \\ & + InhVOL^z_{g(k),t} + InhINV^z_{g(k),t} \end{aligned} \qquad (12)$$

The net labour income obtained by an individual belonging to the class $g(k)$ depends on the fraction of time devoted to working ($l^z_{g(k),t}$), on the wage rate obtained in the period ($w_{lab,t} \cdot A^z_{g(k),t}$), and on the taxation level (where τ_t indicates the direct tax rate, while τ_{cs} indicates the social security rate). $PP^z_{g(k),t}$ indicates the fraction of people in the class $g(k)$ that receive pension benefits, and $Pens^z_{g(k),t}$ indicates the annual value of pension benefits.

The expenditure, indicated by $d^z_{g(k),t}$, are given by the consumption of goods and services and by childbearing expenditure ($c\_childr^z_{g(k),t}$):

$$d^z_{g(k),t} = p_t \cdot c^z_{g(k),t} + c\_childr^z_{g(k),t} \tag{13}$$

In particular, we assume that childbearing expenditure depends proportionally on the number of children ($n\_childr^z_{g(k),t}$) and on the average cost of a child (c^{childr}_t):[13]

$$c\_childr^z_{g(k),t} = n\_childr^z_{g(k),t} \cdot c^{childr}_t \tag{14}$$

Voluntary ($InhVOL^z_{g(k),t}$) and involuntary ($InhINV^z_{g(k),t}$) bequests are uniformly distributed across the different age groups.

Pension benefits are computed using the methods described in the paragraph 4.3.1.

ii) In the case of premature death, the budget constraint is the following:

$$\sum_{k=1}^{T<15} R_{t+k-1} \cdot \left(m^z_{g(k),t+k-1} - d^z_{g(k),t+k-1} \right) \gtreqless 0 \tag{15}$$

Here, the present value of the final wealth for an individual who dies in the age group $g(T)$ (with $T < 15$) may be positive, negative, or null. In this case the final wealth represents an involuntary bequest.

4.1.5 Optimal individual choices

By maximising utility, the individual born in t chooses simultaneously:

i) The fraction of time to devote to schooling, when in the first age group, $g(1)$.

[13] We assume that the average cost of children is independent on the age of the children and that it grows at the PIB growth rate.

ii) The fraction of time to devote to leisure, when he successively belongs to the age groups $g(2)$,..., $g(7)$.

iii) His intertemporal consumption profile.

iv) The amount of bequest to leave if he reaches the age group $g(15)$.

The first order conditions are the following:

i) Decision of studying, which only concerns people born in Italy and belonging to the age group $g(1)$:

$$\begin{aligned} &(1-\tau_t-\tau_c)\cdot\frac{w_{lab,t}\cdot A^{it}_{g(1),t}}{\Delta} \\ =\ &\sum_{k=1}^{9} R_{t+k-1}\cdot(1-\tau_{t+k-1}-\tau_c)\cdot w_{lab,t+k-1}\cdot\frac{\partial A^{it}_{g(k),t+k-1}}{\partial\left[\Delta\cdot\left(1-l^{it}_{g(1),t}\right)\right]} \end{aligned} \tag{16}$$

where R_t represents the capitalisation factor, with $R_{t+k-1}=\prod_{s=t+1}^{t+k-1}\left(\frac{1}{1+r_{net_s}}\right)$, τ_t the tax rate, and τ_c the contribution rate.

This means that if an individual decides at t to study one more year (where $\Delta\cdot\left(1-l^{it}_{g(1),t}\right)$ indicates the number of years devoted to studying by people belonging to the first age group) the individual renounces to one year of wage (the LHS) that, at the optimum, must be equal to the present value of all additional incomes earned thanks to the increase in the productivity related to human capital (the RHS).

ii) Decision concerning the leisure (for the age groups $g(2),...,g(7)$):

$$1-l^z_{g(k),t}=\epsilon_{g(k),t}\cdot\frac{c^z_{g(k),t}}{(1-\tau_t-\tau_c)\cdot w_{lab,t}\cdot A^z_{g(k)}} \tag{17}$$

Everything else equal, an increase in the net wage induces an increase in the individual's labour supply.

iii) Intertemporal profile of consumption:

$$\frac{c^z_{g(k+1),t+1}}{c^z_{g(k),t}}=\frac{\gamma_{g(k+1),t+1}\cdot\left(1+r_{net_{t+1}}\right)}{1+\rho_{g(k+1)}} \tag{18}$$

Therefore, an increase in the survival probability causes, ceteris paribus, an increase in future consumption and in current savings.

iv) The voluntary bequest, indicated by beq_t^z (for the age group $g(15)$):

$$beq_t^z = \beta_{BEQ} \cdot c_{g(15),t}^z \tag{19}$$

The individual's optimal bequest is then proportional to his last period consumption.

4.1.6 Portfolio composition and capital remuneration

The individual wealth owned by the representative individual for the class $g(k)$, indicated by $lend_{g(k),t}^z$, is given by physical capital ($ks_{g(k),t}^z$) and government bonds ($bond_{g(k),t}^z$):

$$lend_{g(k),t}^z = p_{t-1} ks_{g(k),t}^z + p_{t-1} bond_{g(k),t}^z \tag{20}$$

We assume that the portfolio composition is the same for each age group and reflects the structure of the total asset supply.

The rate of return on the existing physical capital (r_{k_t}) - for a period of 5 years - is given by its (net of depreciation) marginal productivity and by the potential capital gain that can be realised if the price of capital at the end of the period (PK_t) is greater then the initial price (PK_{t-1}):

$r_{k_t} = \frac{w_{cap,t} - \delta \cdot PK_t}{PK_{t-1}} + \left(\frac{PK_t}{PK_{t-1}} - 1 \right)$

Individuals can also buy new capital (investment) which is supposed to perfectly substitute the existing capital. Then, the remuneration of new capital is:

$r_{k_t} = \frac{w_{cap,t} - \delta \cdot PK_t}{Pinv_{t-1}} + \left(\frac{PK_t}{Pinv_{t-1}} - 1 \right)$

that implies that the price of the existing capital is equal to the price of the new capital, i.e. $PK_t = Pinv_t$. Furthermore, we consider only one sector in the model, so $PK_t = Pinv_t = p_t$. Consequently, we have that:

$$r_{k_t} = \frac{w_{cap,t} - \delta \cdot p_t}{p_{t-1}} + \left(\frac{p_t}{p_{t-1}} - 1 \right) \tag{21}$$

Physical capital and government bonds are assumed to be perfectly substitutes. Their remuneration is thus the same, excepted for a constant prime risk Π:

$$r_{bond_t} = r_{k_t} - \Pi \tag{22}$$

The interest rate (r_t), that represents the global remuneration of the financial wealth, is given by the weighted average between the remuneration of physical capital and government bonds, where the weights depend on the structure of the total asset supply. The net interest rate is given by $r_{net_t} = r_t \cdot (1 - \tau_t)$.

4.2 The firms

In our economy, only one good is produced using a Cobb-Douglas technology:

$$Y_t = \Gamma \cdot Kd_t^{\alpha} \cdot Ld_t^{1-\alpha} \tag{23}$$

where Y_t represents the production level of the period, Γ the total factor productivity, Kd_t the physical capital demand, and Ld_t the per unit of effective labour demand.

The first order conditions for the profit maximisation are the following:

$$\begin{aligned} w_{cap,t} \cdot Kd_t &= \alpha \cdot pd_t \cdot Y_t \\ w_{lab,t} \cdot Ld_t &= (1 - \alpha) \cdot pd_t \cdot Y_t \end{aligned} \tag{24}$$

where $w_{cap,t}$ and $w_{lab,t}$ represent respectively the gross capital remuneration and the wage rate, and pd_t represents the domestic price.

We assume that the government does not hold physical capital and that the balance of trade is always in equilibrium, so the capital stock is owned by households:

$$Kstock_t = \sum_z \sum_k POP^z_{g(k),t} \cdot ks^z_{g(k),t} \tag{25}$$

The evolution of the capital stock depends on the investments realised in the period (I_t) and on the capital depreciation:

$$Kstock_{t+1} = Kstock_t \cdot (1 - \delta) + I_t \tag{26}$$

4.3 The government

4.3.1 The pension system

The pension system is the first component of the government budget that we consider. The Italian pension system is a Pay-As-You-Go system in which workers pay social security contributions on the basis of 32.7% of wages and retirees receive a pension benefit computed according to the rules introduced by the reforms Amato and Dini. In the model, the value of pension benefits is then computed by applying the earning based method for the pensions paid until 2015, the pro-rata method for the pensions paid between 2020 and 2030, and the contribution based method for the pensions paid from 2035.

For individuals belonging to the age groups $g(8)$ and $g(9)$ (respectively 55-59 and 60-64) the treatment is slightly more complex because in these classes not all individuals work or are retired. We distinguish the two cases.

For the retirees belonging to the age group $g(8)$, pension benefits are computed in the following way:

- Earning based method ($t \leq 2015$): the annual pension benefit is computed on the basis of the average income earned in the 10 last years (last two periods in the model):

$$Pens^z_{g(8),t} = n^z_{g(8)} \cdot 0.02 \cdot \left(\frac{w_{lab,t} \cdot A^z_{g(8),t} + w_{lab,t-1} \cdot A^z_{g(7),t-1}}{2} \right) \tag{27}$$

  where the replacement ratio is proportional to the number of years worked by class $g(8)$, indicated by $n^z_{g(8)}$.

- Contribution based method ($t \geq 2035$): the annual pension benefit is computed by multiplying the transformation coefficient $\beta_{g(8)}$ by the value of the contributions paid during the whole working life and capitalised on the basis of the average GDP growth rate (g_{GDP_t}):

$$Pens^z_{g(8),t} = \beta_{g(8)} \cdot \left(\sum_k \tau_c \cdot w_{lab,t+k-8} \cdot A^z_{g(k),t+k-8} \cdot \prod_{s=t+k-8}^{t} (1 + g_{GDP_s}) \right) \tag{28}$$

  with $1 \leq k \leq 8$ for people born in Italy and $3 \leq k \leq 8$ for immigrants.

- Pro-rata method ($2020 \leq t \leq 2030$): the annual pension benefit is equal to a weighted average between the pension benefit computed with the earning based method and the contribution based method, where the weight depends on the number of years worked before and after 1995.

For the retirees belonging to the age group $g(9)$, we have to consider that only a fraction λ of these individuals retires between 60 and 64 and that the complementary fraction, $1-\lambda$, retires during the previous period (55-59). On average, the pension benefit obtained by the representative individual aged 60-64 is:

- Earning based method ($t \leq 2015$):

$$\begin{aligned} Pens^{z}_{g(9),t} &= \\ & \lambda \cdot \left[n^{z}_{g(9)} \cdot 0.02 \cdot \left(\frac{w_{lab,t} \cdot A^{z}_{g(9),t} + w_{lab,t-1} \cdot A^{z}_{g(8),t-1}}{2} \right) \right] \\ & + (1-\lambda) \cdot Pens^{z}_{g(8),t-1} \end{aligned} \tag{29}$$

- Contribution based method ($t \geq 2035$):

$$\begin{aligned} Pens^{z}_{g(9),t} &= \\ & \lambda \cdot \left[\beta_{g(9)} \cdot \left(\sum_{k} \tau_c \cdot w_{lab,t+k-9} \cdot A^{z}_{g(k),t+k-9} \cdot \prod_{s=t+k-9}^{t} (1 + g_{GDP_s}) \right) \right] \\ & + (1-\lambda) \cdot Pens^{z}_{g(8),t-1} \end{aligned} \tag{30}$$

- Pro-rata method ($2020 \leq t \leq 2030$): with regard to the fraction λ of individuals who retire between 60 and 64, pension benefits are given by a weighted average between the pension benefits computed with the earning based method and the contribution based method, whereas the fraction $1-\lambda$ who retires in the previous period, receives $Pens^{z}_{g(8),t-1}$.

Concerning the indexation of pension benefits, from 1992 onwards, pension benefits are not indexed to real wages, but to the inflation rate. The hypothesis of the money neutrality introduced in the model implies that pension benefits remain constant in real terms over time:

$$Pens^{z}_{g(k),t+k-9} = Pens^{z}_{g(9),t} \tag{31}$$

with $k \geq 10$.

The last problem concerns the determination of the transformation coefficients $\beta_{g(k)}$. These coefficients are fixed by law and vary according to the retirement age of the individual (see Table 30). The transformation coefficients used in the model for the classes $g(8)$ and $g(9)$ are computed by considering the average retirement age inside these two age groups.

The deficit of the pension system ($DEFss_t$) is given by the difference between the pensions paid and the social contributions perceived:

$$\begin{aligned} & DEFss_t && (32) \\ = \; & \sum_z \sum_k POP^z_{g(k),t} \cdot PP^z_{g(k),t} \cdot \Delta \cdot Pens^z_{g(k),t} - \\ & \sum_z \sum_k \tau_{cs} \cdot POP^z_{g(k),t} \cdot l^z_{g(k),t} \cdot w_{lab,t} \cdot A^z_{g(k),t} \end{aligned}$$

where $POP^z_{g(k),t} \cdot PP^z_{g(k),t}$ represents the number of retirees, while $POP^z_{g(k),t} \cdot l^z_{g(k),t}$ represents the number of workers, belonging to the age group $g(k)$.

4.3.2 Public expenditures

In the model we consider three types of public expenditures: those which are related to the education ($Gedu_t$), the health care expenditure ($Gmed_t$) and others public expenditures (G_t) (public security, public administration...).

Public spending on education of young people from 5 to 24 is assumed proportional to the number of people attending school:[14]

$$Gedu_t = \varphi_t \cdot \left[\left(1 - l^{it}_{g(1),t}\right) \cdot POP^{it}_{g(1),t} + POP^{it}_{g(1),t+1} + POP^{it}_{g(1),t+2} + POP^{it}_{g(1),t+3} \right] \quad (33)$$

where $\left(1 - l^{it}_{g(1),t}\right) \cdot POP^{it}_{g(1),t}$ indicates the number of people born in Italy aged from 20 to 24 that study at t, and $POP^{it}_{g(1),t+i}$ (with $i = 1, 2, 3$) the number of people aged from 5 to 19 at t. We also make the assumption that the average expenditure by student (φ_t), varies over time according to the evolution of the GDP.

The health care expenditure is assumed proportional to the number of people aged 60 or more:

$$Gmed_t = \phi_t \cdot \sum_z \sum_{k=9}^{15} POP^z_{g(k),t} \quad (34)$$

We assume that ϕ_t , that represents the average expenditure for people aged over 60, varies over time at the same rate as the GDP[15].

[14] We assume that all individuals from 5 to 19 study.

[15] This is obviously a simplistic representation. However, it is consistent with the health care expenditure projections used by the Italian authorities, which should pass from 5.5% in 1995 to 7.5% in 2050, with respect to GDP.

With regard to the other government expenditures (G_t), we assume they are in fixed proportion with GDP.

4.3.3 Government saving and public debt

Government saving (S_{gov_t}) is given by the difference between revenues (taxes on labour and capital incomes and on pension benefits)[16] and expenditures (on education, on health and others, interests paid on government debt and deficit of the pension system):

$$\begin{aligned} & S_{gov_t} \qquad (35) \\ = \; & \tau_t \cdot \sum_z \sum_k POP^z_{g(k),t} \cdot \left[l^z_{g(k),t} \cdot w_{lab,t} \cdot A^z_{g(k),t} + r_t \cdot lend^z_{g(k),t} + PP^z_{g(k),t} \cdot \Delta \cdot Pens^z_{g(k),t} \right] \\ - \; & \left[p_t \cdot (Gedu_t + Gmed_t + G_t) + r_t \cdot p_{t-1} \cdot B_t + DEFss_t \right] \end{aligned}$$

We make the assumption that the ratio between the public debt and GDP remain constant over time, so:

$$\frac{p_{t-1} \cdot B_t}{pd_t \cdot Xs_t} = const \qquad (36)$$

where B_t indicates the number of public bonds.

The equation (36) determines the level of public savings that permits to keep the ratio of the public debt to GDP constant, while the equation (35) determines the direct tax rate (τ_t) that permits to respect this budget constraint.

The evolution of the public debt is given by the following dynamic equation:

$$p_t \cdot B_{t+1} = p_{t-1} \cdot B_t - S_{gov_t} \qquad (37)$$

4.4 The rest of the world

The quantity of goods that is available in the market (X_t) is given by goods produced in Italy that are not exported (Xxd_t) and by imports (M_t). To this composite good (X_t) is associated an aggregate price p_t, while the price of domestic sales and the world price are respectively pd_t and pw_t. We consider the Armington hypothesis that domestic goods and foreign goods are not perfect substitutes because of their different geographic origin.[17]

[16] Government revenues also include indirect taxes on production and on consumption. These taxes are modelled but they are not indicated in the equations to ease notation.

[17] The Armington hypothesis has been introduced in the OLG models by Bettendorf and Heijdra (2001).

The first order conditions that determine the optimal level of domestic sales, of imports, and of the aggregate price are the following:

$$Xxd_t = \alpha_{Xxd} \cdot \left(\frac{p_t}{pd_t}\right)^{\sigma_M} \cdot X_t \tag{38}$$

$$M_t = \alpha_M \cdot \left(\frac{p_t}{pw_t}\right)^{\sigma_M} \cdot X_t \tag{39}$$

$$p_t \cdot X_t = pd_t \cdot Xxd_t + pw_t \cdot M_t \tag{40}$$

with $X_t = \sum_z \sum_k POP^z_{g(k),t} \cdot c^z_{g(k),t} + (G_t + Gedu_t + Gmed_t) + I_t$, i.e. the composite good is used as consumption good for households and for the government, and as investment good.

The demand by the rest of the world for the domestic good is given by a decreasing function of the domestic price:

$$E_t = E_{0t} \cdot \left(\frac{pw_t}{pd_t}\right)^{\sigma_E} \tag{41}$$

where we assume that E_{0t} grows at the GDP growth rate.

We also suppose that the balance of trade is always in equilibrium, so:

$$Pd_t \cdot E_t = Pw_t \cdot M_t \tag{42}$$

4.5 The equilibrium conditions

i) Market of goods and services: the production level must be equal to the domestic and foreing demand:

$$Y_t = Xxd_t + E_t \tag{43}$$

ii) Labour market: the demand of per unit of effective labour must be equal to the quantity supplied by the workers:

$$Ld_t = \sum_z \sum_k POP^z_{g(k),t} \cdot l^z_{g(k),t} \cdot A^z_{g(k),t} \tag{44}$$

iii) Market of physical capital: the demand of physical capital by firms must be equal to the quantity supplied by the household:

$$Kd_t = Kstock_t \tag{45}$$

iv) Market of government bonds:

$$B_t = \sum_z \sum_k POP^z_{g(k),t} \cdot Bond^z_{g(k),t} \tag{46}$$

The equilibrium conditions determine the equilibrium level of prices, i.e. the domestic price (pd_t), the wage per unit of effective labour ($w_{lab,t}$), the capital remuneration ($w_{cap,t}$) and the remuneration on government bonds (r_{bond_t}). The *numéraire* good is represented by the foreign good, so we fix $pw_t = 1$, and one equilibrium condition is redundant because of the Walras law.

5 Calibration of the model

The model is calibrated "out of the steady state" using data of the year 1995. The goal of our calibration is to reproduce, after the introduction of the demographic change[18] and the introduction of the pension system reforms of the Nineties, the economic situation of 1995.

This method of calibration differs from the methods generally used (see, for example, Hviding and Mérette (1998)): these models are generally calibrated at the steady state with a constant population (or with a constant population growth) and a constant productivity growth rate, by using a set of macroeconomic data of the Fifties and Sixties; then, demographic shock introduced in the model. By the consequence, the simulation results of the demographic shock do not properly reproduce the macroeconomic data of the latest years. By contrast, the model presented in our study is calibrated in 1950 in a way that - after the introduction of the demographic shock and of the pension system reforms - the solution of the model for the year 1995 reproduces the 1995 Italian macroeconomic data (in particular, the value of the GDP, the ratio between aggregate consumption and GDP, the ratio between investments and GDP, and the ratio between public expenditure and GDP).

Moreover, the calibration of the model also allows to reproduce the propensities to save of the different age groups in 1995 (in order to correctly evaluate the evolution of

[18] The demographic shock is simulated by introducing into the model the exogenous values of fertility rates, mortality rates and immigration flows determined to reproduce as closely as possible demographic projections by Istat.

the capital supply), and finally to replicate the most important ingredients of the pension system.

Concerning the calibration of the propensities to save, in the overlapping-generations models, the profiles of consumption are calibrated, while the profiles of saving are residual. Consequently, and in application of the life-cycle theory, the active adults present a high propensity to save while old people a negative propensity to save. The population ageing would thus cause a strong reduction in the national saving. The microeconomic analyses on propensities to save of the different age groups (Poterba (1994)) suggest different conclusions, actually inconsistent with the life-cycle theory: the oldest classes have positive propensities to save, possibly very high as for example in Italy and in Japan. The macroeconomic effects of ageing on the aggregate saving would be in this case appreciably different, and in particular weaker, compared to those measured by the earlier models.

In order to correctly evaluate the evolution of the capital supply, we thus chose to calibrate propensities to save for the different age groups so that we can reproduce the microeconomic data available. To achieve this goal, we calibrated the intertemporal profile of consumption and the value of the voluntary bequest left at the end of the last period of life. Consequently, we obtain an intertemporal preference rate calibrated (ρ) different for each age group. That enables us to correctly reproduce the saving profile of the different age groups, which would have been impossible by considering a single value of ρ.[19] In addition, we decided to use an intertemporal preference rate that is different for each age group because with the introduction of mortality rates, the effective intertemporal preference rate varies according to the age of the individual, because it depends on its survival probability.

Bequests play also a fundamental part in the calibration of propensity to save of the different age groups. If the heritage is not taken into account (such as for example in the model of Miles (1999)), old people are obliged to consume entirety, or almost, of their wealth before the end of the last period of life, so they certainly present negative values of propensities to save which are not coherent with the real data.

[19] In the majority of the OLG models, ρ lies between 0.5% and 1.5% and is definitely lower than the interest rate. The profile of consumption will be consequently increasing and exponential and thus incompatible with the real data.

With regard to the modelling of the pension system, it is necessary to consider the characteristics of the Italian system. First of all, the pension benefits can be classified in three categories, on the basis of the nature of the risk and the need that the pension system seeks to ensure:[20]

i) Old-age function, to cover the risk related to the ageing of the individual (loss of income, insufficient income...).

ii) Disability function, to cover the risk of incapacity to carry on an activity of work.

iii) Reversibility function, to cover the economic risk in case of spouse dead.[21]

Concerning the old-age function, which interests us in this analysis, we separated the old-age pensions from the social aid. Then, we took into account the basic pensions paid by the public institutions. Tables 10 to 15, in Appendix, give information concerning the number of pensions and the pension system expenditures for the year 2000. We are specifically interested in the aggregate constituted by the basic pensions paid by the public institutions to the pensioners over 55. These pension benefits are perceived by the employees of the public administration, the employees of the private companies and the self-employed workers. In order to simplify the analysis, we made the assumption that all the workers belong to the same system, i.e. the social contributions rate and the calculation rule of the pension benefits are the same for all workers.[22]

The goal of this calibration is to reproduce, for the aggregate we defined, the ratio between the number of pensioners and the number of workers, which was 0.667 in 2000, and the ratio between the pension expenditure and the GDP, which was 10.8% in 2000.

Table 16 shows the values of the calibrated parameters used in the model, whereas Table 17 indicates the values of some endogenous variables produced by the model and

[20] Istat (2003), "Statistiche della previdenza e dell'assistenza sociale. I trattamenti pensionistici. Anni 2000-2001".

[21] The pension benefits are included in this category only if the retiree is aged less than 65; if not, the pension benefit is included in the old-age function.

[22] In reality, the contribution rate applied for the employees of the public administration (32%) is very close to that applied for the workers of private companies (32.7%); on the other hand, the contribution rate applied for self-employed workers is substantially lower (15.6%).

compared to the 1995 data. In particular, for equation (4) which determines the individual productivity level related to his age, the parameters θ, θ_1 and θ_2 are calibrated in order to reproduce, by a quadratic regression, the earning profile used by Hviding and Mérette (which predicts the maximum value at 52 years). The parameter α_{HC} in equations (5) is calibrated in order to reproduce the data concerning the fraction of people aged 20-24 who study and we applied an annual human capital depreciation rate of 1%. The parameter χ in equation (7) is calibrated in order to reproduce an annual economic growth rate of about 2%. The parameters $\rho_{g(k)}$ and β_{BEQ} in the utility function (equation (9)), which respectively represent the intertemporal preference rates and the intensity of the preference for bequests, are calibrated in order to reproduce the data concerning the propensities to save of the different age groups in 1995.

The parameters $\epsilon_{g(k),t}$, which represent the intensity of the preference for leisure, are calibrated in order to reproduce the data concerning the occupational rates of the different age groups in 1995. These parameters vary over time in order to take into consideration the increase of the activity participation of the women which depends both on the evolution of wages and on cultural factors.

The childbearing expenditure is calibrated by considering, for the year 2000 that a cost for a child (see equation (14)) represents, on average, 30% of the parent consumption. Concerning the production function, the part of the capital remuneration in the GDP was 52.2% in 1995 . We used an annual depreciation rate of the physical capital of 2%.

In 1995 the national debt accounted for 120% of the GDP. In 1997 the public spending on education accounted for 3.8% of the GDP and in 2000 the expenditure on public health accounted for 5.5% of the GDP. The parameter φ_t and ϕ_t, respectively in equations (33) and (34), are calibrated in order to reproduce these ratios.

In order to guarantee the convergence of the economy towards a balanced growth path, the structure of the population is supposed to be constant beyond a certain date (35 periods in the model, i.e. 175 years): the fertility rates and the survival probabilities are then supposed to be constant and the immigration is null.

Both the calibration and simulations of the model were made by using numerical algorithms provided by GAMS (General Algebraic Modelling System).

6 Macroeconomic effects of ageing

In this section, we analyse the simulated macroeconomic effects of the Italian demographic evolution and of the introduction of the Amato and Dini reforms, i.e. the indexation of the pension benefits on inflation and the modification of the calculation rule of the pension. This projection is compared to the one that would be obtained if these reforms were not adopted, in order to evaluate their efficiency.

Figure 18 shows that the population ageing phenomenon considerably depresses the ratio between investments and GDP during the period 2005-2040. In particular, in the presence of the pension system reforms, this ratio passes from 19.6% in 1995 to 12.8% in 2050. That is due to several elements. First, the pension system reform causes a strong fall in the value of the pension benefits perceived from 2020 onwards.[23] In order to maintain unchanged the consumption profile, the generations that will undergo this reduction modify their consumption behaviours and increase their saving during the work period in order to accumulate more capital to spend during the following period when their incomes will be lower. Therefore, the propensities to save of these generations during the retirement age will be lower than the propensities to save of previous generations. Second, since the pension system will produce considerable deficits in the next decades, it will be necessary to increase the taxation level. The evolution of the tax rate (Figure 20) causes a reduction of the net incomes and thus of the savings for each age group. Another element is given by the evolution of the fertility rates. As shown in Table 6, the fertility rates fall considerably during the first time and increase since 1995. This evolution influences the childbearing expenditure and thus the savings of households with children. Figure 19 indicates the evolution of the propensities to save for the age groups 35-39, 45-49, 55-59 and 65-69, in the presence of the pension system reforms.

Nonetheless, if the reforms were not adopted, it would have been necessary to strongly increase the general tax rate (Figure 20) to maintain unchanged the ratio of public debt to GDP. The negative effect on the aggregate savings would have been considerably amplified and the ratio of the investments to GDP would drop from 17.2% in 1995 to 7.3% in 2050

[23] In fact, the pro-rata method is applied from 2020 onwards and the new method is applied from 2035 onwards.

(Figure 18). From this point of view, the reforms thus seem effective.

The pension system reforms allow also a favourable evolution of the GDP and of the per capita GDP. Figure 21 indicates the annual GDP growth rate in the presence and in the absence of reforms. However, even in presence of the reforms, we can note that starting from 2005, the evolution of the GDP growth rate is decreasing and that from 2025 onwards it will be lower than 2%, i.e. the level of the productivity growth. That is due, on one hand, to the reduction of the investment rate, and, on the other hand, to the strong reduction of the active population.

Concerning the evolution of the per capita GDP (Figure 22), the reduction of the annual growth rate between 2010 and 2030 is due to the reduction of investment ratio as well as to the fact that the population ageing which causes an increase in the unproductive fraction of the population. From 2030 onwards, the per capita GDP growth rate increases due to the favourable evolution of the occupational rate (Figure 23). The evolution of the price of the production factors is reported in Figures 24 and 25. The population ageing, and thus the reduction in the active population, causes an important increase in the wage per unit of effective labour. The evolution of the average capital remuneration depends on the increase of the per worker capital supply.

The important changes on future wages and interest rates will cause significant impacts on the decision to invest in human capital. In Figure 26, we note that the pension system reforms allow a more favourable evolution of the fraction of time devoted to studies over the period taken into account. In particular, by considering the introduction of the reforms, we can note an increase until 2025 of this index due to the reduction of the interest rate and thus to the increase in the present value of the future wages. After 2025, young people are less incited to study, because of the maintenance of the interest rates and wages at a constant level. The effect on the productivity growth rate related to the level of knowledge present in the economy (Figure 27) is always positive, in particular in the case of introduction of the reforms. This result is similar with that of the model of Fougère and Mérette (1999): the population ageing, by causing an increase in wages, create opportunities for young people to invest in human capital, which stimulates the economic growth.[24]

[24] In Fougère and Mérette (1999) the human capital investment has a very positive effect on the economic

7 Effects of ageing on the pension system

The impacts at the macroeconomic level on the pension system are represented in Figures 28 and 29 which respectively indicate the evolution of the total expenditure and of the deficit of the pension system compared to GDP. These evolutions clearly state that the impact of the population ageing on the pension system, in the absence the Amato and Dini reforms, would have been disastrous. Nevertheless, we can note that the introduction of the reforms is very effective only during the first time (1995-2010), thanks to indexation of pension benefits to inflation. After this period, the strong increase in the dependency ratio causes an important deterioration of the financial situation of the pension system, phenomenon worsened by the long duration of the transitional period envisaged by the Dini reform: in 2040, the ratio between the pension expenditure and the GDP will be equal to 13.8% and the deficit of the pension system compared to the GDP will be equal to 4.5%. The financial situation will improve only since 2040, but the deficit of the pension system will remain considerable: in 2055 it will be about 1.7% of GDP.

It is also important to analyse the effects of the Dini reform on the pension benefits perceived by individuals according to different retirement ages and by considering the three methods for the calculation of the pension benefits: the earning based method, for those who in 1995 had more than 18 years of contributions; the pro-rata method, for those who in 1995 had less than 18 years of contributions; the contribution based method, for those who started to work after 1995. Consequently, the earning based method is applied until 2015, the pro-rata method is applied since 2020, and the contribution based method is applied since 2035.

We analyse two indices: the replacement ratio - the ratio of the gross pension to the

growth. By contrast, in our model the increase in the productivity growth rate (g_H) is less important: between 1995 and 2045, it passes from 1.99% to 2.34%. However, we want to emphasise that this result is coherent with the empirical data. Barro (2001) estimated, over the period 1965-1995, that the increase of one year devoted to studies by individuals aged more than 25 years increases the economic growth rate of 0.44%. In our model, the average number of years devoted to studies by individuals who work passes from 2.9 in 1995 to 3.73 in 2045. According to the estimation of Barro, this increase in the time devoted to studies should produce an increase in the annual productivity growth rate of 0.365%, which is very close to the increase obtained by our model (+0.35%).

last gross salary perceived - and the implicit rate of return on contributions.

We note that, with respect to the partial equilibrium analysis, the interest rates, the wage growth rate (used to determine the level of social contributions) and the GDP growth rate (used to capitalise the contributions with the *pro-rata* method and the contribution based method) are not constant, but they are endogenously generated by the model.

Table 31, that reports the evolution of the replacement ratios, shows the reduction in the value of pension benefits perceived by all age groups when the contribution based method is applied.[25] Clearly, the replacement ratio is not an appropriate index to evaluate the equity level of a pension system. In this respect, we have to consider that, for example, people who retire one year later pay one additional year of contributions and renounce to one year of pension benefits. Therefore, the implicit rate of return on contributions is more appropriate than the replacement ratio.

The implicit rate of return on contributions, indicated by RI, is defined as the interest rate that equalises the capitalised value of contributions paid to the actualised value of pension benefits perceived:

$$\sum_{t=N}^{T} C_t \cdot (1 + RI)^{T-t} = \sum_{t=T+1}^{85} P \cdot (1 + RI)^{T-t} \tag{47}$$

with $57 \leq T \leq 64$. Here, N represents the beginning of working activity, T the last period of work, C_t the yearly flow of social contributions, P the yearly flow of pension benefits (constant over time, since pension benefits are indexed to inflation, see equation (31)). The rate of return on contributions is computed for an individual who works without interruptions and who receives the pension benefits until 85 years.

Table 32 shows the existence until 2005 of a strong difference between people who retire at 57 with respect to people who retire later. The fact of advancing the departure to the retirement is undoubtedly profitable. In 2005, i.e. with the application of the earning based method, the implicit rate of return on contributions was 3% for people who retire at 57 and 1.4% for people who retire at 64. This represents a situation of severe iniquity and, on the other hand, an incitation for early retirement which is very negative for the economic system.

[25] The unique exception concerns people who retire at 63 and 64 during the application of the *pro-rata* method.

The introduction of the contribution based method permits a reduction of the inequality level. In 2035 the difference between the implicit rate of return on contributions for people who retire at 57 and for people who retire at 64 would be 0.59%, against 1.65% in 2005.

8 Sensibility analysis

In this section we present two sensibility analysis. The first one evaluates the effects on the pension system by considering different productivity growth rates. Until now, we used a value for the parameter χ in equation (7) that gives a productivity growth rate equal to 2% in 1995. We now compare the results of the base case with the results of two different scenarios: the first one (Scenario A) in which we consider a lower productivity level and the second one (Scenario B) in which we consider a greater productivity level. The value of the parameter χ is modified from 2005 onwards in order to obtain a productivity growth rate equal to 1.5% for the Scenario A and equal to 2.5% for the Scenario B.

The evolution of the GDP growth rate, that is used in the capitalisation of social contributions with the new method of computation of the pension benefits, is indicated in Figure 33.

It is straightforward that a greater economic growth determines a better financial situation for the pension system. A greater economic growth induces, on one hand, an increase in social contributions perceived and, on the other hand, an increase in pension benefits paid, because of the increase in wages and in the capitalisation rate. Given that the increase in revenues is greater than the increase in expenditures, the total effect on the pension system is positive.

In particular, in the case of a favourable productivity growth (Scenario B), the ratio of the pension system deficit to GDP (Figure 34) would be equal to 3.5% in 2040 (*vs.* 4.5% in the Base Case and 5.1% in the Scenario A) and it would be equal to 0.7% in 2055 (*vs.* 1.7% in the Base Case and 2.3% in the Scenario A). Table 35 also shows that a greater productivity growth rate implies a greater rate of return on contributions, due to the application of the contribution based method.

The second sensibility analysis concerns the occupational rate. In the Base Case, we have reproduced the evolution of the occupational rate by considering the projections made by the Brambilla ministry commission[26] for the period 2000-2050: the activity rate would increase from 59.6% to 67.8% and the unemployment rate would decrease from 10.6% to 4.5%. This implies that the occupational rate would increase from 53% in 2000 to 65% in 2050. The increase in the occupational rate is mainly due to a strong increase in the labour participation of women (from 46.4% to 59.9%) that essentially depends on cultural factors rather than economic ones.

With the second sensibility analysis we analyse the effects on the pension system of a less optimistic evolution of the occupational rate. The results of the Base Case are now compared to those obtained with the hypothesis that the occupational rate only depends on economic factors, i.e. on the wage evolution (Scenario C). The evolution of the occupational rate is shown by Figure 36. The reduction in the occupational rate has a negative impact on the financial situation of the pension system (Figure 37). In 2040, the ratio of the pension system deficit to GDP would be 5.2% *vs.* 4.5% in the Base Case, and in 2055 it would be 2.4% *vs.* 1.7%.

The negative impact on the pension system determined by the reduction in the number of workers is partly compensated, on one hand, by the increase in wages and then by the increase in contributions perceived, and in the other hand, by the decrease in pension benefits paid. In fact, the reduction in the occupational rate induces a reduction in the GDP growth rate and, consequently, in the implicit rate of return on contributions (Table 38).

9 Conclusions

In this paper we analyse the effects of ageing of the Italian population and evaluate the efficiency of Amato and Dini reforms. A pension system reform was necessary for two main reasons: to avoid the financial bankrupt of the pension system and to reduce the negative effects on the economic growth given by the important reduction in national savings and, consequently, in investments.

[26] Relazione della Commissione Ministeriale per la valutazione degli effetti della legge n.335/95 e successivi provvedimenti (2001).

In this analysis we used an overlapping-generations general equilibrium model. In fact, the general equilibrium approach allows not only to evaluate the macroeconomic effects (the impacts on the aggregate consumption, aggregate savings, national production, wages, interest rates...), but also the effects on the pension system, since the wage evolution affects the evolution of the social security contributions, while the GDP growth rate affects the value of pension benefits according to the Dini reform.

The macroeconomic effects of the introduction of the pension system reforms seem to be positive, since they reduce the negative impact on the national savings, thus, a better evolution of GDP and per capita GDP.

Another positive aspect of the reforms is that they will penalise early retirement. Before the reforms, the implicit rate of return on contributions for people who retire at 57 was strongly greater with respect to people who retire later.

Nevertheless, we show that the reforms are not sufficient. They allow avoiding an enormous increase in the ratio between the total expenditure and the GDP. Even with an optimistic hypothesis related to the productivity growth and to the evolution of women activity rates, the pension system equilibrium will be not reached. Moreover, the deficit of the pension system will attain, for the period 2025-2045, 3-5% of GDP. For these reasons, a new reform seems to be inevitable.

In this study, we also show the problems related to the equity at the intergenerational level induced by the Dini reform. Given that the new reform applies different criteria on the basis of the age of individuals in 1995, it strongly advantages people who in 1995 have more than 18 years of contributions, since for them the method of computation of the pensionbenefits has not been modified; in contrast, the reform penalises the next generations that, when the reform was introduced, had not election rights.

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Appendix A

Data

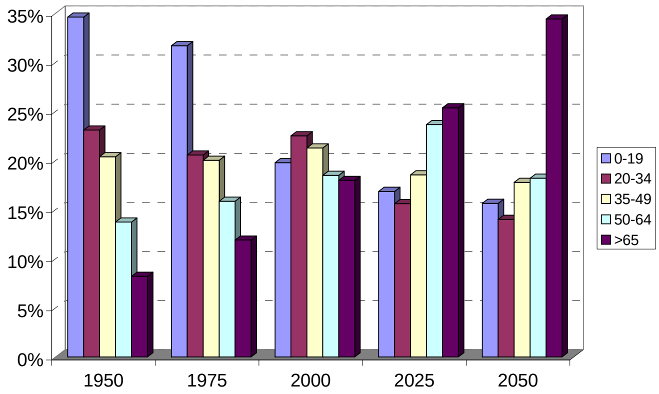

Figure 1: Evolution of the structure of the population (in % of the population)

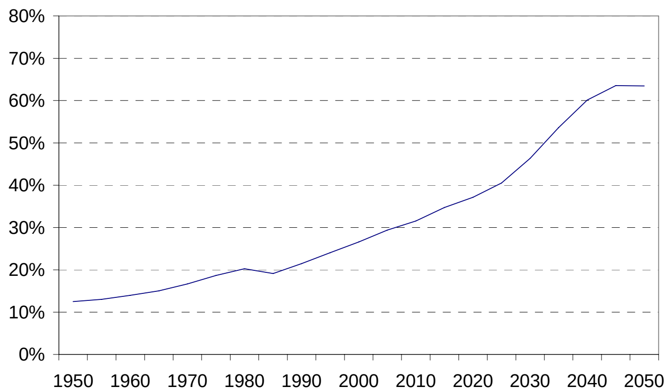

Figure 2: Evolution of the old-age dependency ratio (>65 / 15-64)

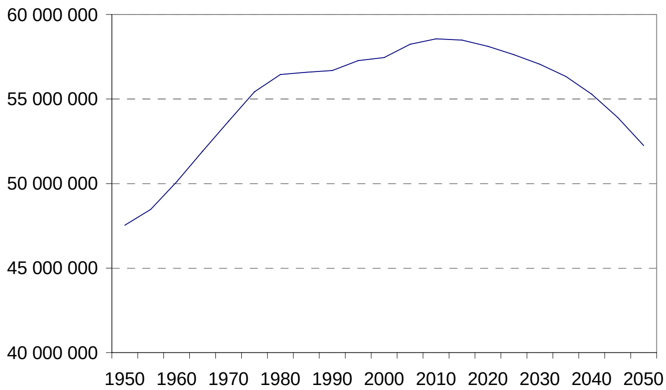

Figure 3: Evolution of the total population

20-24	99.73%
25-29	99.69%
30-34	99.68%
35-39	99.61%
40-44	99.45%
45-49	99.17%
50-54	98.65%
55-59	97.86%
60-64	96.59%
65-69	94.49%
70-74	90.86%
75-79	84.84%
80-84	75.58%
85-89	59.57%
90-94	43.63%
95-99	26.24%
100-104	12.13%
105-109	4.21%
110-114	1.05%
115-119	0.19%

Table 4: Survival probabilities (Istat, 2000)

	g(10) 65-69	*g(11)* 70-74	*g(12)* 75-79	*g(13)* 80-84	*g(14)* 85-89	*g(15)* >90
1950	66.6%	64.6%	61.5%	51.2%	45.0%	35.0%
1955	67.0%	72.3%	61.9%	52.6%	45.4%	35.4%
1960	69.0%	80.3%	69.9%	60.6%	47.1%	35.8%
1965	72.9%	81.8%	77.8%	68.6%	52.6%	36.2%
1970	80.9%	82.2%	78.2%	69.0%	53.0%	37.0%
1975	88.3%	82.6%	78.6%	69.4%	53.4%	37.4%
1980	90.8%	85.1%	79.0%	69.8%	53.8%	37.8%
1985	93.3%	87.6%	81.5%	72.3%	56.3%	40.3%
1990	93.7%	90.1%	84.0%	74.8%	58.8%	42.8%
1995	94.1%	90.5%	84.4%	75.2%	59.2%	43.2%
2000	94.5%	90.9%	84.8%	75.6%	59.6%	43.6%
2005	94.9%	92.1%	86.3%	77.1%	61.1%	45.1%
2010	95.3%	92.5%	87.8%	78.6%	62.6%	46.6%
2015	95.7%	92.9%	89.2%	80.1%	64.1%	48.1%
2020	96.1%	93.3%	89.6%	81.6%	65.6%	49.6%
2025	96.5%	93.7%	90.0%	83.1%	67.1%	51.1%
2030	96.9%	94.1%	90.4%	84.2%	68.1%	52.6%
2035	97.3%	94.5%	90.8%	84.6%	68.5%	53.2%
2040	97.3%	94.5%	90.8%	84.6%	68.5%	53.2%
2045	97.3%	94.5%	90.8%	84.6%	68.5%	53.2%
2050	97.3%	94.5%	90.8%	84.6%	68.5%	53.2%

Table 5: Survival probabilities used in the model for people aged more than 65

	20-24	25-29	30-34	35-39
1950	0.235	0.492	0.442	0.174
1955	0.212	0.442	0.392	0.156
1960	0.192	0.400	0.342	0.144
1965	0.208	0.434	0.377	0.152
1970	0.206	0.430	0.382	0.159
1975	0.192	0.381	0.351	0.153
1980	0.167	0.331	0.301	0.131
1985	0.142	0.281	0.251	0.109
1990	0.105	0.231	0.201	0.079
1995	0.082	0.181	0.151	0.060
2000	0.105	0.221	0.184	0.073
2005	0.128	0.268	0.242	0.104
2010	0.126	0.275	0.253	0.110
2015	0.126	0.275	0.253	0.110
2020	0.126	0.275	0.253	0.110
2025	0.126	0.275	0.253	0.110
2030	0.126	0.275	0.253	0.110
2035	0.135	0.283	0.261	0.114
2040	0.160	0.335	0.309	0.135
2045	0.205	0.429	0.395	0.172
2050	0.216	0.452	0.416	0.181

Table 6: Fertility rates used in the model

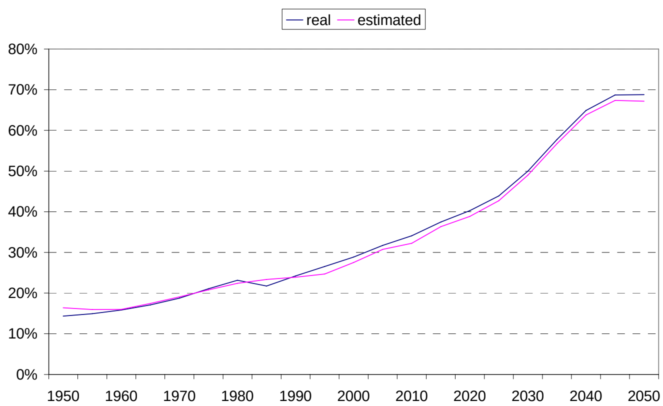

Figure 7: Reproduction in the model of the old-age dependency ratio (>65 / 20-64)

a) age group 20-34

b) age group 35-49

c) age group 50-64

d) age group >65

Figure 8: Reproduction in the model of the demographic evolution for the different age groups

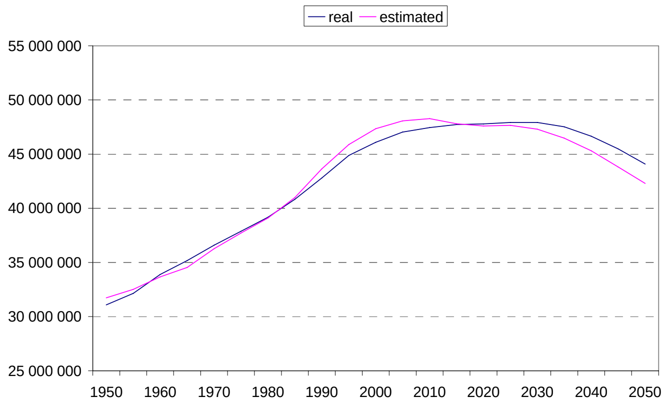

Figure 9: Reproduction in the model of the total population aged more than 20 years

	Public Institutions	Private Institutions	Total
Pensions	13 901 000	382 011	14 283 011
basic pensions	13 800 959	126 273	13 927 232
complementary pensions	100 041	255 738	355 779
Social aids	3 966 962		3 966 962
Total	17 867 962	382 011	18 249 973

Table 10: Old-age function (number of pensions)

	Public Institutions	Private Institutions	Total
Workers in public administration	2 269 194		2 269 194
Workers in the private sector	9 082 022	4 710	9 086 732
Entrepreneurs	2 425 978		2 425 978
Liberal professions	23 765	121 563	145 328
Total	13 800 959	126 273	13 927 232

Table 11: Old-age function, basic pensions (number of pensions)

	Public Institutions	Private Institutions	Total
Workers in public administration	36 789	0	36 789
Workers in the private sector	79 736	204	79 940
Entrepreneurs	15 174	0	15 174
Liberal professions	125	1 353	1 478
Total	131 823	1 558	133 381

Table 12: Old-age function, basic pensions (expenditure in millions of euros)

	Number of pensions	Expenditure
40-44	8 947	98
45-49	72 356	944
50-54	344 655	5 184
55-59	965 202	14 555
60-64	2 057 639	23 890
65-69	2 972 263	26 883
70-74	2 683 508	23 235
75-79	2 209 190	18 345
80-84	1 150 345	9 037
85-89	919 978	6 713
90-94	340 414	2 387
95 +	74 120	529
autre	2 342	23
Total	13 800 959	131 823

Table 13: Old-age function, basic pensions, public institutions

	Number of pensions	Population	# pens. / population
55-59	965 202	3 388 966	28.5%
60-64	2 057 639	3 396 644	60.6%
65-69	2 972 263	3 112 130	95.5%
70-74	2 683 508	2 740 398	97.9%
75-79	2 209 190	2 251 253	98.1%
80-84	1 150 345	1 048 244	109.7%
85-89	919 978	846 356	108.7%
>90	416 876	325 648	128.0%
Total	13 375 001	17 109 639	

Table 14: Old-age function, basic pensions, public institutions, more than 55 years

Number of pensions	Expenditure	# pensions / # workers	Expenditure / GDP
13 375 001	125 597	66.0%	10.78%

Table 15: Old-age function, basic pensions, public institutions, more than 55 years

HOUSEHOLDS		
Productivity related to the age	θ	1.234
	θ_1	0.284
	θ_2	-0.019
Productivity related to the education	α_{HC}	0.117
Productivity related to the average level of knowledge	χ	0.059
Intertemporal elasticity of substitution		1
Index of preference for leisure	$\varepsilon_{g(2)}$	0.600
	$\varepsilon_{g(3)}$	0.451
	$\varepsilon_{g(4)}$	0.351
	$\varepsilon_{g(5)}$	0.296
	$\varepsilon_{g(6)}$	0.301
	$\varepsilon_{g(7)}$	0.411
Index of preference for bequest	β_{BEQ}	3.026
FIRMS		
Depreciation rate of physical capital	δ	10.4 %
Capital remuneration in the added value	α	52.2%
GOVERNMENT		
Contribution rate	τ_c	32.7 %
Public debt / GDP		120 %
Total public expenditure / GDP		16 %

Table 16: Values of some parameters used in the model

	Simulated value	Real value
Direct tax rate	15.2 %	
GDP (in milliards of euros)	923.053	923.052
Consumption / GDP	61.84 %	60.6 %
Investments / GDP	19.64 %	19.3 %
Gedu / GDP	3.88 %	3.8 %
Gmed / GDP (in 2000)	5.54 %	5.5 %
Pensions / GDP (in 2000)	11.3 %	10.8 %
Retirees / Workers (in 2000)	0.652	0.66
Propensities to save $s_{g(3)}$	19.8 %	20 %
$s_{g(4)}$	25.6 %	26 %
$s_{g(5)}$	21.7 %	22 %
$s_{g(6)}$	22.4 %	23 %
$s_{g(7)}$	30.5 %	31 %
$s_{g(8)}$	31.7 %	32 %
$s_{g(9)}$	33.3 %	34 %
$s_{g(10)}$	35.6 %	36 %
$s_{g(11)}$	30.8 %	31 %
Occupational rates $l_{g(1)}$	35.9 %	35.9 %
$l_{g(2)}$	57.1 %	57.8 %
$l_{g(3)}$	68.1 %	68.8 %
$l_{g(4)}$	72.5 %	72.0 %
$l_{g(5)}$	71.2 %	71.4 %
$l_{g(6)}$	67.1 %	67.2 %
$l_{g(7)}$	54.8 %	55.4 %
$l_{g(8)}$	37.7 %	37.7 %
$l_{g(9)}$	18.2 %	18.2 %
National occupational rate	54.62 %	54.57 %

Table 17: Values in 1995 of some endogenous variables

Appendix B

Results of simulations

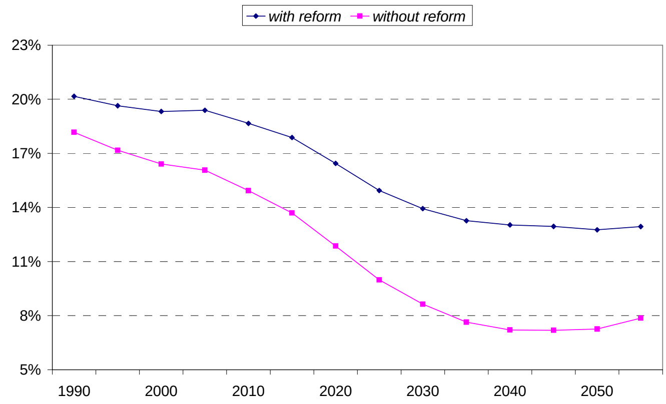

Figure 18: Investments / GDP

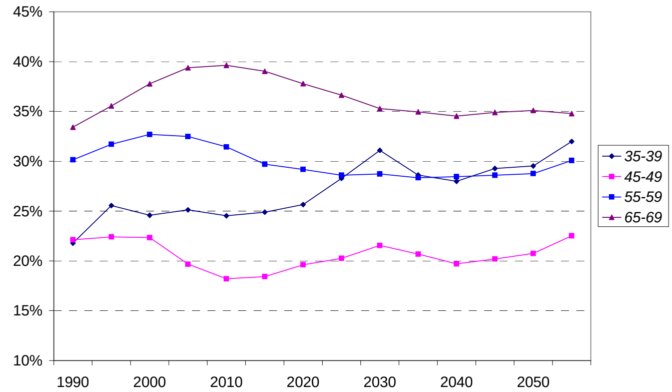

Figure 19: Propensity to saving for some age group (with reform)

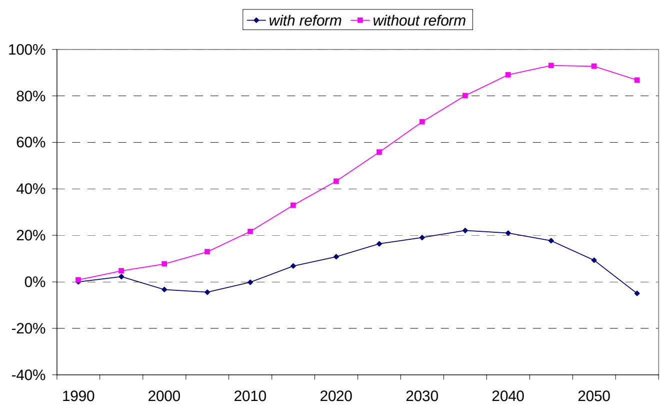

Figure 20: Direct tax rate (% variations with respect to year 1990)

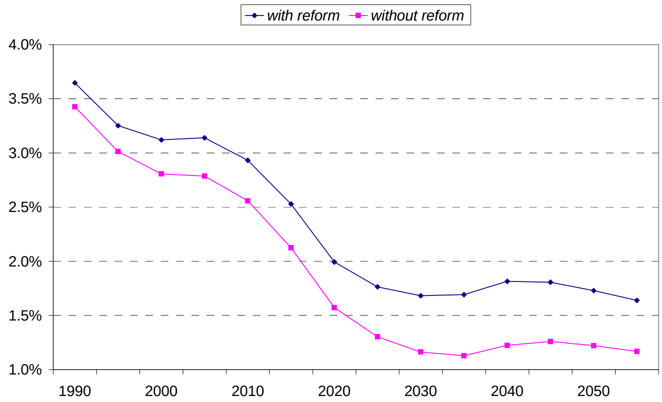

Figure 21: GDP growth rate

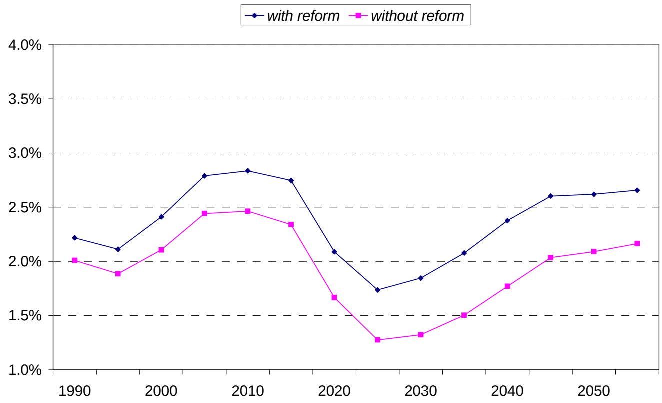

Figure 22: per capita GDP growth rate

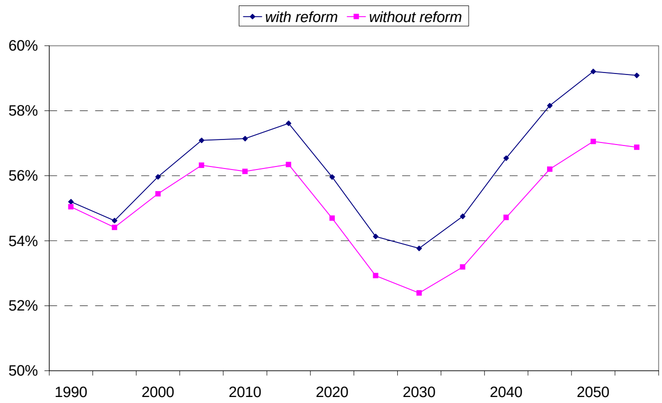

Figure 23: Occupational rate

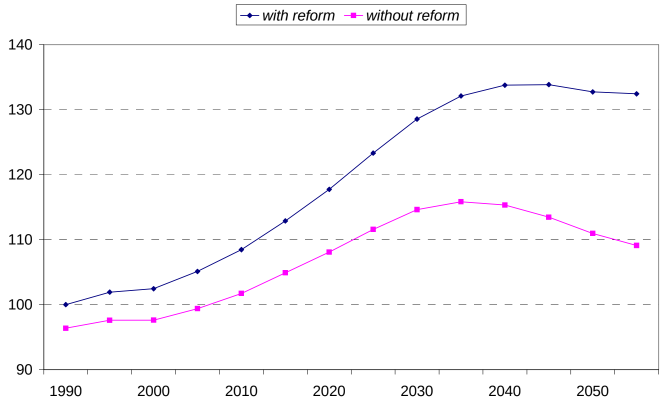

Figure 24: Wage per unit of effective labour (wage in 1990 = 100)

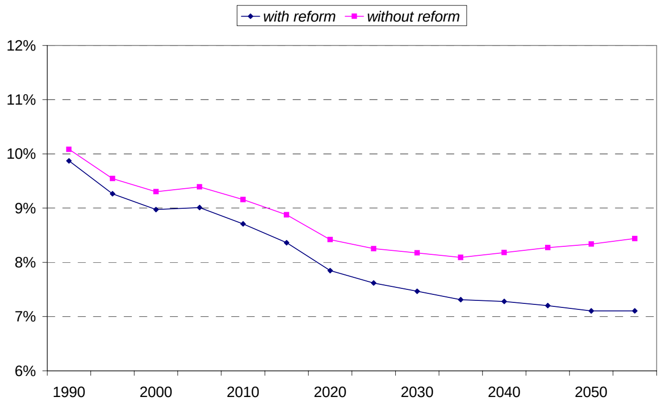

Figure 25: Interest rate

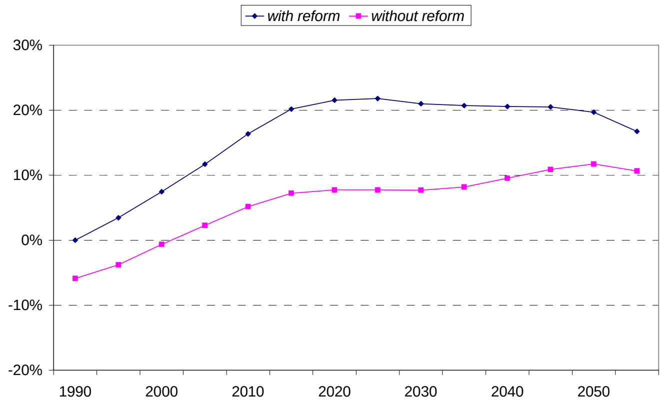

Figure 26: Time devoted to schooling (% variations with respect to year 1990)

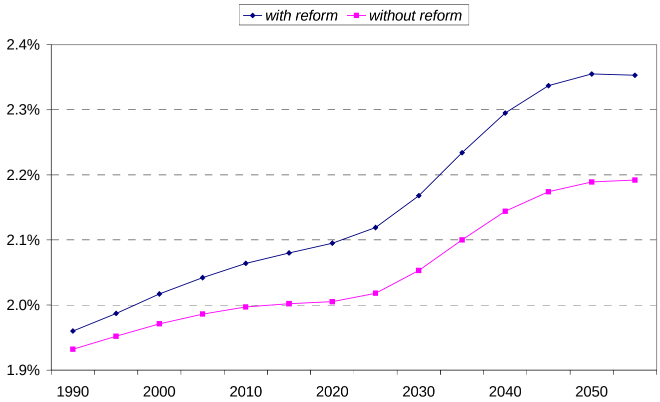

Figure 27: Productivity growth rate (g_H)

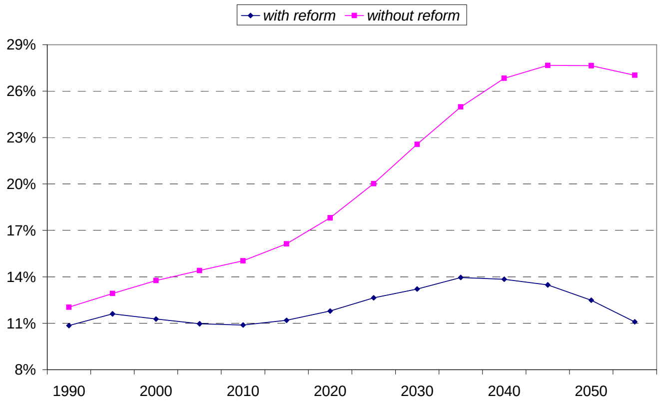

Figure 28: Pensions / GDP

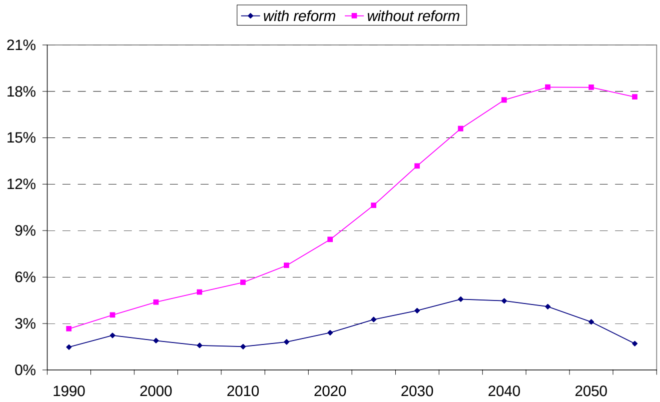

Figure 29: Pension system deficit / GDP

Retirement age	
57	4.720%
58	4.860%
59	5.006%
60	5.163%
61	5.334%
62	5.514%
63	5.706%
64	5.911%

Table 30: Transformation coefficients

Retirement age	57	58	59	60	61	62	63	64
Years of contribution	35	36	37	38	39	40	41	42
2005	69.2%	71.2%	73.2%	75.0%	77.0%	79.0%	81.0%	82.9%
2010	69.8%	71.7%	73.7%	75.7%	77.6%	79.6%	81.6%	83.6%
2015	70.3%	72.3%	74.3%	76.4%	78.4%	80.5%	82.5%	84.5%
2020	61.2%	64.6%	68.1%	72.3%	76.1%	79.9%	83.9%	88.0%
2025	56.6%	60.3%	64.1%	68.7%	72.8%	77.0%	81.4%	85.9%
2030	51.2%	55.1%	59.1%	64.2%	68.6%	73.1%	77.8%	82.6%
2035	49.6%	52.9%	56.4%	61.0%	65.1%	69.5%	74.1%	79.1%
2040	48.6%	51.9%	55.3%	59.9%	63.9%	68.2%	72.7%	77.6%
2045	48.8%	52.0%	55.4%	60.0%	64.0%	68.2%	72.8%	77.6%
2050	49.4%	52.6%	56.1%	60.7%	64.7%	68.9%	73.5%	78.4%
2055	50.1%	53.4%	56.8%	61.5%	65.5%	69.9%	74.5%	79.4%

Table 31: Replacement ratios

Retirement age	57	58	59	60	61	62	63	64
Years of contribution	35	36	37	38	39	40	41	42
2005	3.06%	2.84%	2.61%	2.35%	2.13%	1.90%	1.66%	1.41%
2010	3.12%	2.90%	2.66%	2.39%	2.17%	1.94%	1.70%	1.45%
2015	3.20%	2.97%	2.74%	2.47%	2.24%	2.01%	1.76%	1.51%
2020	2.70%	2.58%	2.44%	2.30%	2.17%	2.03%	1.87%	1.71%
2025	2.44%	2.35%	2.25%	2.14%	2.04%	1.92%	1.78%	1.64%
2030	2.09%	2.04%	1.98%	1.92%	1.85%	1.76%	1.65%	1.53%
2035	1.99%	1.91%	1.82%	1.75%	1.67%	1.59%	1.50%	1.40%
2040	1.89%	1.82%	1.74%	1.67%	1.59%	1.51%	1.42%	1.33%
2045	1.85%	1.78%	1.70%	1.63%	1.56%	1.48%	1.40%	1.30%
2050	1.84%	1.77%	1.69%	1.62%	1.55%	1.47%	1.39%	1.30%
2055	1.83%	1.76%	1.68%	1.61%	1.54%	1.46%	1.38%	1.29%

Table 32: Implicit rate of return on contributions

Appendix C

Sensibility analysis on the productivity level

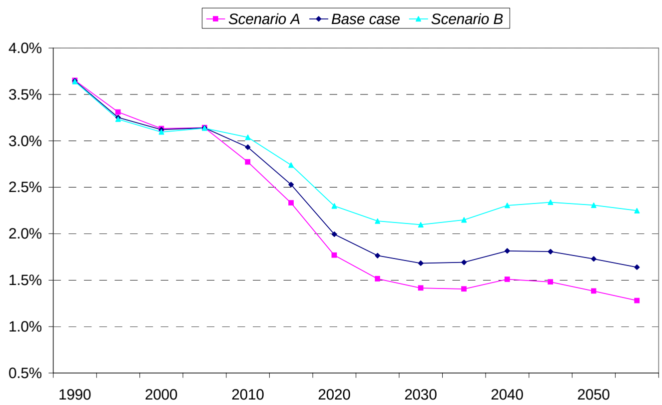

Figure 33: GDP growth rate

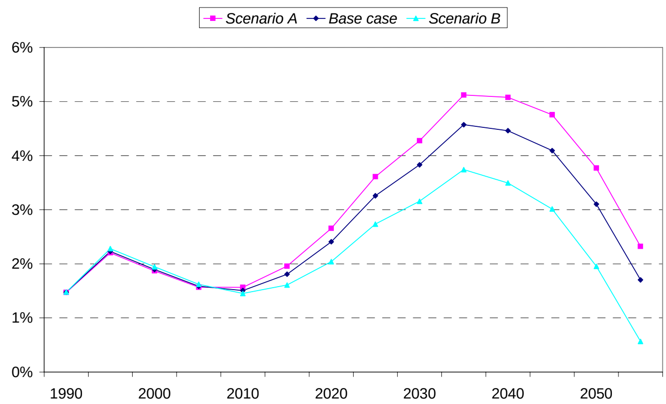

Figure 34: Pension system deficit / GDP

Retirement age	57			60			64		
	Scenario A	*Base case*	*Scenario B*	*Scenario A*	*Base case*	*Scenario B*	*Scenario A*	*Base case*	*Scenario B*
2025	2.36%	2.44%	2.60%	2.07%	2.14%	2.28%	1.59%	1.64%	1.75%
2030	1.99%	2.09%	2.29%	1.83%	1.92%	2.10%	1.47%	1.53%	1.67%
2035	1.87%	1.99%	2.24%	1.65%	1.75%	1.97%	1.32%	1.40%	1.57%
2040	1.76%	1.89%	2.18%	1.55%	1.67%	1.91%	1.24%	1.33%	1.52%
2045	1.71%	1.85%	2.17%	1.50%	1.63%	1.90%	1.20%	1.30%	1.51%
2050	1.68%	1.84%	2.18%	1.48%	1.62%	1.91%	1.19%	1.30%	1.53%
2055	1.66%	1.83%	2.19%	1.46%	1.61%	1.93%	1.17%	1.29%	1.54%

Table 35: Implicit rate of return on contributions

Appendix D

Sensibility analysis on the occupational ratio

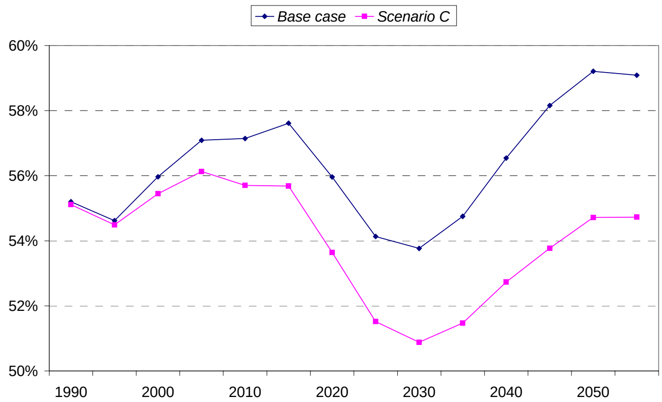

Figure 36: Occupational ratio

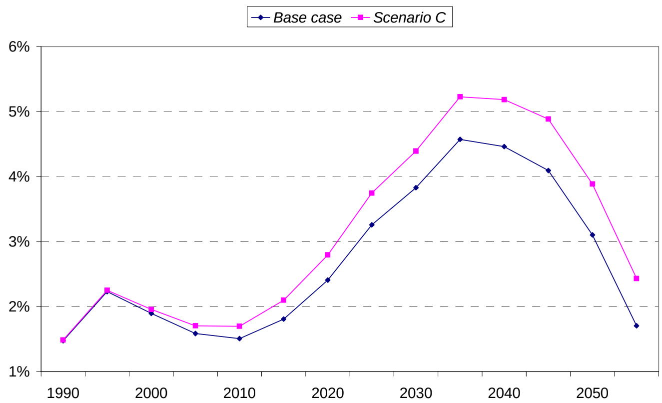

Figure 37: Pension system deficit / GDP

Retirement age	57		60		64	
	Base case	*Scenario C*	*Base case*	*Scenario C*	*Base case*	*Scenario C*
2025	2.44%	2.38%	2.14%	2.10%	1.64%	1.61%
2030	2.09%	2.01%	1.92%	1.86%	1.53%	1.49%
2035	1.99%	1.92%	1.75%	1.69%	1.40%	1.35%
2040	1.89%	1.82%	1.67%	1.60%	1.33%	1.28%
2045	1.85%	1.79%	1.63%	1.57%	1.30%	1.25%
2050	1.84%	1.77%	1.62%	1.56%	1.30%	1.24%
2055	1.83%	1.76%	1.61%	1.55%	1.29%	1.24%

Table 38: Implicit rate of return on contributions

Retirement age, immigration or pension benefits? An applied general equilibrium evaluation of a pension reform in an ageing context (the Italian case)

Abstract

Most European countries have recently introduced pension system reforms to face the financial problem related to population ageing. Italy is no exception. The reforms introduced during the Nineties are generally thought not sufficient to adequately face the population ageing problem. The Berlusconi government has introduced in 2004 a new reform that increases the retirement age. Using an applied overlapping-generations general equilibrium model with endogenous growth due to human capital accumulation, we analyse the impact of this reform on the macroeconomic system and in particular on the pension system. Then, we evaluate the impacts of complementary reforms - an immigration policy and the reduction in pension benefits - that could be set up in order to achieve the lung-run equilibrium of the pension system.

1 Introduction

The pension system reforms introduced during the Nineties, the Amato reform in 1992 and the Dini reform in 1995, are unanimously regarded as being insufficient in the medium-run - because of the implied long transition phase which will produce important social security deficits - as well as in the long-run: as shown by Magnani (2006), even when completely applied, the reforms cannot be expected to achieve the financial equilibrium of the pension system. In that paper, we also have shown that the effects on the macroeconomic system are likely to be negative: the reduction in pension benefits and the resulting increase in taxes necessary to face the pension system deficits, will induce a fall in national savings, reduce capital accumulation and slow down economic growth. As a consequence, a new pension system reform seemed inevitable: in 2004, the Berlusconi government decided to increase the retirement age from January 2008 onwards. Other European countries have adopted, or consider adopting, similar reforms in order to face the population ageing problem.

Our first objective in this paper is to evaluate the Berlusconi reform, that is to evaluate its impact on the macroeconomy and on the pension system of the increase in the retirement age. We show that this reform would induce a very important reduction in pension deficits in the medium-run, but it becomes completely ineffective in the long-run. We then explore some complementary policies which again are on the agenda of numerous other European countries: one is immigration. Indeed, immigration is often thought to provide an alternative to pension system reforms, since young immigrants permit to reduce the old-age dependency ratio. In this ageing context, there is emerging debate in European countries on the virtues of increasing immigration.[1]

We then investigate whether, in the Italian context, a more favourable and selective migration policy, complementary to the Berlusconi reform, could solve the long-run financial problem. We conclude that the increase in the number of yearly immigrants necessary

[1] The conclusion that seems to predominate in the literature is that migration can alleviate but not counter the demographic shock. A partial equilibrium analysis by the European Commission and Eurostat (2002) suggests that even doubling immigration and fertility rates will not be sufficient to compensate the increase in the old-age dependency ratio and then to guarantee a significant contribution to securing sustainable pension systems.

to achieve the pension system's long-run financial balance is too high to be politically feasible. It seems therefore that, unpopular as such reform may be, a reduction in pension benefits is necessary to reach the lung-run equilibrium of the pension system.

Our assessment is based on simulation exercises using an applied overlapping-generations general equilibrium model. A dynamic general equilibrium perspective is indeed required in order to evaluate the impacts on the macroeconomy and on the pension system, since population ageing will significantly affect future labour supply (and thus the evolution of wages) and capital accumulation (and thus the evolution of investments, interest rates and GDP). The evolution of wages directly affects the evolution of the social security contributions, whereas the evolution of the GDP growth rates affects the evolution of pension benefits since, with the Dini reform, pension benefits are computed on the basis of the contributions that are paid during the whole working life and that are capitalised at the GDP growth rate.

Another important aspect related to the demographic change and to the introduction of a pension reform is the impact on education decisions and consequently on economic growth. Indeed, relative factor prices are likely to vary significantly in the next decades hence affecting the decision to invest or not in human capital. One can expect that the impact of population ageing on human capital formation will be positive, since ageing will boost labour wages and reduce interest rates, and that the increase in retirement age will encourage individuals to devote more time to schooling. The positive impact on economic growth could be important[2] and, consequently, could produce positive effects on the financial situation of the pension system by increasing future wages and social contributions.

The model we use is an applied overlapping-generations general equilibrium model of the type pioneered by Auerbach and Kotlikoff (1987), though with significant differences: we introduce mortality, immigration, human capital accumulation, and endogenous growth. The introduction of mortality and immigration makes it possible to accurately reproduce the Istat's demographic projections and to simulate the effects of changes in immigration policy. The introduction of human capital makes it possible to introduce a

[2] Barro (2001) estimates that an additional year of schooling by people aged 25 and older induces an increase in the economic growth rate by 0.44% per year.

mechanism of endogenous growth based on the average level of knowledge present in the economy. Human capital results from explicit decision making by young people to invest time in education.

The paper is organised as follows: in the next section, we describe the main characteristics of the OLG model. In sections 3, we describe the "out of steady state" calibration. Section 4 presents the results of the Berlusconi reform in terms of impacts on the macroeconomy and on the pension system. Sections 5 and 6 present the results of the simulations concerning the immigration policies and the reduction in pension benefits. We draw our conclusions in the last section.

2 The characteristics of the model

The model presented in this paper is an applied overlapping-generations general equilibrium model in which 15 age groups (20-24, 25-29, ..., 90-94) coexist at each period. The full description of the model is included in Magnani (2006).

The model includes immigration. At the end of each period, people belonging to the last age group die, a fraction of people belonging to the other classes dies, and a new generation enters the active population. We assume that the fertility rates and the survival rates are identical for people born in Italy and immigrants[3], and that immigration is limited to the age group 30-34.[4]

We reproduce the demographic projections presented by Istat[5] for the period 1950-2050 by using the survival rates presented by Istat (2000) for the first 9 age groups, while the survival probabilities for the other age groups and the fertility rates have been calibrated in order to reproduce the Italian demographic evolution. We adopt migratory flows between 100,000 and 120,000 individuals per year from 1990 onwards, following the Istat's assumptions.

[3]Mayer and Riphahn (1999) estimated that the fertility rates of immigrants tend to converge to the fertility rates of the natives.

[4]This assumption, that allows us an important simplification of the model, is justified by the fact that data concerning resident permits (Istat, 2004) are normally distributed with a peak for the age group 30-34. In any case, the introduction in the model of immigration at different age does not significantly change the results.

[5]Istat (2001), Previsioni della popolazione residente per sesso, età e regione. Base 1.1.2001.

The quality of the calibration of demographic variables to Istat's projections is summarised in Figure 1: we report the old-age dependency ratio, which represents the most important demographic variable; we see that the quality of the fit is high.

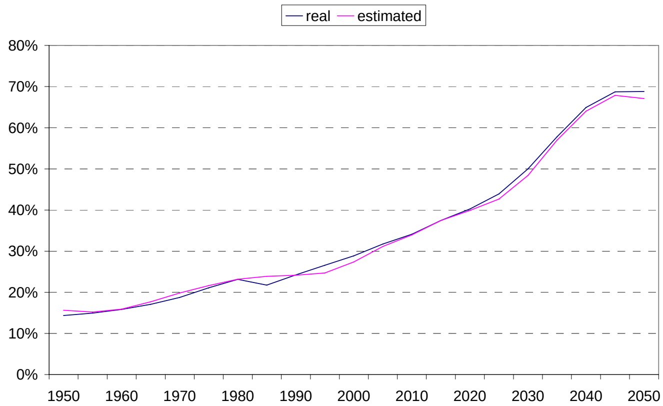

Figure 1: Reproduction in the model of the old-age dependency ratio (>65 / 20-64)

For each age group we assume that there exists a representative agent of people born in Italy and a representative agent of immigrants (intra-generation's heterogeneity), that agents have perfect foresight and that there is no liquidity constraint.

Individuals maximise an intertemporal utility function subject to an intertemporal budget constraint. Immigrants and people born in Italy have the same structure of preferences. They decide the intertemporal profile of consumption and leisure as well as the value of the voluntary bequest that will be left at the end of the last period of life. On the other hand, only people born in Italy decide the fraction of time to devote to studying. This decision allows the individual to constitute a stock of human capital that affects his productivity level and, consequently, his future earning profile.

Intra-generation's heterogeneity is given by the assumption that immigrants differ from people born in Italy by a lower level of productivity[6] and that they enter in Italy with no capital. On the other hand, the children of immigrants are considered identical to the children of people born in Italy. Consequently, they must decide the fraction of time to devote to studying and the difference in productivity disappears.

[6] Storesletten (2000) estimates for the United States that the productivity of people who immigrate at 37 years old is lower by 13% with respect to the natives.

People who die in the last period (95 years old) decide to leave a bequest to the other generations, on the basis of a maximisation process of their utility function: in this case, there are voluntary bequests. On the other hand, people belonging to the other age groups, in the case of premature death, do not program the value of their final wealth: in this case, there are involuntary bequests. Voluntary and involuntary bequests are uniformly distributed among the other generations.[7]

In our economy, only one good is produced using a Cobb-Douglas technology. Labour and capital markets are assumed perfectly competitive, so real wages and real interest rates adjust to equilibrate aggregate demand and aggregate supply.

Aggregate capital supply depends on the individual's capital accumulation, while aggregate labour supply depends on the demographic evolution and on the individual's choice about the amount of time devoted to working. In this model, people belonging to the first 9 age groups work and labour supply is supposed endogenous for the first 7 age groups. In particular, people belonging to the first age group (20-24 years old) must decide the fraction of time to devote to the human capital formation. The following age groups, until the class 50-54, must decide the fraction of time to devote to working and to leisure. With regard to the two last age groups who work (55-59 and 60-64), the fraction of people who work is exogenously fixed, according to the 1995 data. This permits us to simulate the impact of an exogenous increase in the retirement age.

We introduce an endogenous growth mechanism based on human capital accumulation *à la* Lucas (1988).[8] Labour income earned by an individual is given by the product between the wage per unit of effective labour and the individual's total productivity level. The wage per unit of effective labour, that is the same for each individual, is endogenously determined in order to guarantee the labour market equilibrium. On the other hand, the individual productivity level is given by the sum of three elements: the productivity related to his age, the productivity related to his education level, i.e. to the fraction of time devoted to schooling in the first period of life (20-24), and the productivity related to the average level of knowledge in the economy, that is supposed to grow at an endogenous

[7] One can avoid involuntary bequests by introducing an insurance mechanism (Yaari, 1965).

[8] Other OLG models that include an endogenous growth mechanism are provided by Fougère and Mérette (1999) and Bouzahzah et al. (2002).

rate depending on the weighted average of the stocks of human capital accumulated by each class that coexists at the same period.

2.1 The pension system

The Italian pension system is almost entirely composed of a compulsory public system that is financed as a Pay-As-You-Go system. An important anomaly of the Italian pension system is that there is not a clear separation between the pension system in the strict sense and a system of social aids, in which benefits are not related to contributions. In particular, the Italian pension system includes pensions related to work (old-age pensions, disability pensions, pensions paid in the case of occupational diseases and industrial injuries), and other pensions (survival pensions, and welfare benefits for people over 65 lacking adequate means of support). Moreover, until 1992, the Italian pension system was characterised by a very large number of funds and schemes, in which contributions and benefit rules varied according to the sector (private or public sector, or self employment). Since our objective is to evaluate the impact on the pension system in the presence of population ageing, we only take into account the old-age pensions.

Workers pay social security contributions on the basis of 32.7% of wages and retirees receive a pension benefit computed according to the rules introduced by the reforms Amato and Dini. In particular:

- For people who started working after 1995, pension benefits are computed according to the *contribution based method*: the contributions paid during the whole working life are virtually capitalised at the GDP growth rate; the value of the pension is equal to the capitalised value of the contributions multiplied by a transformation coefficient (β) which depends on the retirement age.

- For people who in 1995 had more than 18 years of contributions, pension benefits are computed according to the *earning based method*, i.e. on the basis of the average of the labour incomes earned during the 10 last years.

- For people who in 1995 had less than 18 years of contributions, pension benefits are computed according to the *pro-rata method*: the pension is equal to a weighted average between the pension computed with the earning based method and the contribution based method.

Consequently, the value of pension benefits is computed in the model by applying the earning based method for the pensions paid until 2015, the pro-rata method for the pensions paid between 2020 and 2030, and the contribution based method for the pensions paid from 2035.

For individuals belonging to the age groups 55-59 and 60-64 the treatment is slightly more complex because in these classes not all individuals work or are retired. In particular, for the retirees belonging to the age group 60-64, we have to consider that only a fraction of these individuals retires between 60 and 64 and that the complementary fraction retires during the previous period (55-59).

Concerning the indexation of the pension benefits, from 1992 onwards, pension benefits are not indexed to real wages, but to the inflation rate, and therefore remain constant in real terms over time.

Moreover, with the Dini reform, individuals can retire aged 57 or more with at least 5 years of contributions, or with 40 years of contributions. The goal of this reform is to penalise early retirement because, with the contribution based method, if an individual works less, the value of pension benefits will be lower since he accumulates a lower value of contributions and the transformation coefficient applied will also be lower.

Given that these pension system reforms are insufficient to face the population ageing problem, in 2004 the Berlusconi government has introduced a new reform that increases the retirement age. Whereas with the Dini reform workers can decide to retire between 57 and 65, with the Berlusconi reform the minimum retirement age is increased to 60 after January 2008 (61 for the self-employed workers) and to 61 after 2010 (62 for the self-employed workers). In 2012 the government will decide whether or not to increase the minimum retirement age once more: from 2015 it could become 63.

3 Calibration of the model

The aim of our calibration is three-fold: to reproduce the 1995 Italian macroeconomic data (in particular, the value of GDP, the ratio between aggregate consumption and GDP, the ratio between investments and GDP, and the ratio between public expenditure and GDP), to reproduce the propensities to save of the different age groups, and finally to replicate the most important ingredients of the pension system:[9] that is, the ratio of the number of retirees to the number of workers, and the ratio of the total pension expenditure to GDP.

The model is calibrated conditional on the demographic change, on an annual productivity growth rate of about 2%, and on the pension reforms of the 90's. In particular, the demographic change is introduced through a combination of changes in fertility rates, mortality rates and immigration flows, determined to reproduce as closely as possible demographic projections by Istat.

The details about the calibration procedure are indicated in Magnani (2006).

4 Effects of the Berlusconi reform

We now use the model to evaluate the pension reform introduced in 2004 by the Berlusconi government. The effects of this reform is analysed by two simulations: in the first (referred to as *BERL1*) the minimum retirement age is imposed at 60 after 2008 and at 61 after 2010; in the second simulation (*BERL2*) the minimum retirement age is further delayed to 63 years after 2015. The results are contrasted with projections without the Berlusconi reforms, our base case.

4.1 Macroeconomic impacts of the Berlusconi reform

First of all, the increase in the retirement age will have an impact on the labour supply. Figures 2 and 3 in Appendix A show that, with respect to the base case, the increase in the retirement age induces an increase in the occupational rate, i.e. the ratio between the number of workers to the active population, and a reduction in the ratio of the number

[9] As we have already mentioned, we consider in the model only old-age pensions. In particular, we consider the basic pensions paid by the public institutions to the retirees over 55, not the complementary pensions (ISTAT (2003), Statistiche della previdenza e dell'assistenza sociale. I trattamenti pensionistici. Anni 2000 - 2001; table 5.3). We consider a representative agent for each age group and therefore we assume that all workers within each cohort belong to the same pension system, i.e. they pay the same social security contribution rate and they receive a pension benefit computed with the same rule.

of retirees to the number of workers.[10] The increase in the labour supply causes a fall of wages with respect to the base case (Figure 4) that induces individuals to substitute leisure to work. In addition, from 2030 onwards, the base scenario presents a rate of growth of the number of workers higher than the scenarios with the increase in the retirement age (Figure 5).

The increase in the retirement age, and then in the overall lifetime spent working, affects positively the individual time devoted to studying (Figure 6). The greater investment in human capital formation induces a greater pace of the productivity growth rate (Figure 7), with respect to the base case.[11] This positive effect is produced from 2030 onwards since the productivity growth rate depends on the weighted average of the productivity levels of each agent.

The macroeconomic effects, in terms of economic growth and capital accumulation, of delayed retirements are positive until 2035. Initially, the reform has a favourable impact on the ratio of investments to GDP (Figure 8). This is because, as we will see in the next section, the reform strongly reduces the pension system deficit until 2040. This permits to reduce the tax level[12] (Figure 9), with a positive impact on savings. The rise in the occupational rate together with higher capital accumulation boosts GDP growth (Figure 10) and per capita GDP growth (Figure 11). After year 2040, however, GDP and per capita GDP growth rates are very similar in the three scenarios: in fact, in the Berlusconi reform with respect to the base case, the positive effect on productivity growth rate is compensated by the reduction in the rate of growth of the number of workers (Figure 5).

[10] Note that, from an economic point of view, the ratio of the retirees to the workers speaks more than the old-age dependency ratio, since it also considers the evolution of the occupational ratio.

[11] Note that our results are consistent with empirical data. Barro (2001) estimates that an additional year of schooling by people aged 25 and older raises the growth rate by 0.44% per year. In the base case, for example, the average number of year of schooling, in the period 2005-2055, increases by 0.6 (from 3.0 in 1995 to 3.6 in 2055). By considering the Barro estimation, the annual growth rate would have to increase by 0.264%, that is close to the increase predicted by our model (+0.3%, from 2.02% to 2.32%).

[12] The tax rate is endogenously computed to ensure a constant ratio of public debt to GDP.

4.2 Effects of the Berlusconi reform on the pension system

Figures 12 and 13 in Appendix A report on the evolution of the pension system. We see that initially the increase in the retirement age has a very positive impact on the financial situation of the pension system, both in terms of the deficit and of the aggregate expenditure, with respect to GDP. The milder reform (*BERL1*) makes it possible to reduce the ratio of the deficit to GDP of about 1.4% in 2010, 1.3% in 2025 and 0.4% in 2035. As could be expected, an additional increase of the retirement age to 63 after 2015 (*BERL2*) reinforces these effects: -1.4% in 2010, -1.9% in 2025 and -0.9% in 2035.

On the other hand, in the long-run the increase in the retirement age is completely ineffective. As we can see in Figure 17, in year 2040, the *BERL1* reform results in the same ratio of the pension system deficit to GDP as in the base case, and from 2045 onwards, this ratio increases above its base case level. The situation is only slightly better with the *BERL2* reform: the reform is ineffective in reducing pension system deficits beyond year 2045 and the deficit gets worse than in the base case after that date.

In order to understand why the reform becomes ineffective, we have to consider that an increase in the retirement age induces, for the age groups concerned by the reform, a present loss (represented by the additional contributions paid and by the foregone pension benefits) and future gains (represented by the increase in the value of the pension thereafter).[13]

[13] The increase in pension benefits depends on the level of the rate of return on contributions. The rate of return on contributions is defined as the rate that equalises the expected capitalised value of the contributions paid and the expected present value of the pensions obtained: $\sum_{t=N}^{T} C_t \cdot (1+R)^{T-t} \cdot \Omega_t = \sum_{t=T+1}^{X} P \cdot (1+R)^{T-t} \cdot \Omega_t$, where N represents the beginning of working activity, T the last period of work, X the maximum death age, C_t the yearly flow of social contributions, P the yearly flow of pension benefits (constant over time, since pension benefits are indexed to inflation), Ω_t the probability that an individual is alive in t, and R the yearly rate of return on contributions.

Suppose now that the individual decides (or is constrained by the law) to work one more year, i.e. until $T+1$. In order to simplify the analysis, we consider the case in which the rate of return on contributions does not change. Let ΔP be the increase in pension benefits. Given the definition of the rate of return on contributions, we have: $\sum_{t=N}^{T+1} C_t \cdot (1+R)^{T+1-t} \cdot \Omega_t = \sum_{t=T+2}^{X} [P + \Delta P] \cdot (1+R)^{T+1-t} \cdot \Omega_t$. After some mathematical manipulations, we find that $C_{T+1} \cdot \Omega_{T+1} + P \cdot \Omega_{T+1} = \sum_{t=T+2}^{X} \Delta P \cdot (1+R)^{T+1-t} \cdot \Omega_t$, i.e. the expected present value of the future increase in pension benefits ΔP is equal to the sum of the additional one-year contributions that he pays and the pension that he gives up. The increase in pension

It is clear that, in the early years of the reform, the increase in the retirement age can only have favourable effects on pension system. However, as time passes, a larger number of individuals receive the increase in pension benefits, so the reform ceases to be effective. To show this, let us imagine that before the reform each individual retires at 58 and that the reform increases the retirement age by one year from 2008 onwards. In 2008 people forced to postpone retirement pay one more year of contributions and loose one year of pension benefits. Their loss represents a net gain for the pension system, since pension benefits do not change for any age groups that year. The next year, the pension system receives the same increase in contributions but this gain is now partially compensated by the increase in pension benefits paid to the retirees that, without the reform, would have retired at 58 in 2008 and, with the reform, retire at 59 in 2009. In 2010 two age groups benefit from the increase in pensions: people who, without the reform, would retire at 58 in 2008 and in 2009 but, with the reform, are constrained to work one additional year. Etc. The Appendix D formalises this mechanism. Such a partial equilibrium calculation suggests that a reform that increases the retirement age by one year stops being effective at year 2038.

Another element that makes the reform ineffective is related to the contribution based method introduced by the Dini reform in order to penalise early retirement. As Table 14 shows, in 2005 the rate of return on contributions for those who retire at 57 is largely higher than that of individuals who postpone retirement; in contrast, from 2040 onwards, the difference between the rates of return on contributions reduces significantly. This means that, when the earning based method is applied, if an individual works one more year, the increase in the value of his pension is less important than in the case in which the rates of return on contributions are equal for all individuals.[14] In contrast, with the

benefits is therefore $\Delta P = \frac{\Omega_{T+1} \cdot \left(C_{T+1}+P\right)}{\sum_{t=T+2}^{X} (1+R)^{T+1-t} \cdot \Omega_t}$.

[14] Consider an individual who retires in T, with a rate of return on contributions equal to R_1, so $\sum_{t=N}^{T} C_t \cdot (1+R_1)^{T-t} \cdot \Omega_t = \sum_{t=T+1}^{X} P \cdot (1+R_1)^{T-t} \cdot \Omega_t$.

Suppose now that the individual decides to work one more year, i.e. until $T+1$. The increase in pension benefits depends on the level of the rate of return on contributions.

Consider first the case in which the rate of return on contributions does not change. As already seen (note 13), the expected present value of the future increase in pension benefits, indicated by ΔP_1, is equal to the additional contributions that he pays as well as the pension that he gives up: $C_{T+1} \cdot \Omega_{T+1} + P \cdot \Omega_{T+1} =$

contribution based method and the presence of an actuarial link between pension benefits and contributions paid, if an individual decides to work one more year, the increase in the value of his pension is more relevant. As a consequence, the increase in the retirement age causes an increase in pension benefits that is more important when the contribution based method is applied. So, the fact that from 2045 onwards the majority of the retirees receive a pension computed with the contribution based method represents another element that influences negatively the evolution of pension system.

4.3 Generational accounting

Finally, we analyse for each generation the gains and the losses related to the Berlusconi reform by using the generational accounting approach introduced by Auerbach et al. (1994). As we can see in Table 15 in Appendix A, the analysis begins with the generation born in 1935, which becomes active in 1955 and retires in 1993. For each generation, we compute the ratio of the expected present value of the revenues (pension benefits and per capita government expenditure) to the expected present value of the payments (direct taxes and social security contributions). In the base case, we consider a representative individual who stops working at 58. In the simulation *BERL1*, we consider until year 2003 an individual who stops working at 58, and beyond 2011 one who works until age 61. In addition, in simulation *BERL2*, we consider an individual who stops working at 63 after year 2018.

The results of this analysis are shown in Figure 16. First of all, by considering the base case, we note that the value of this index decreases starting from the generation born in 1960 because the introduction of the pro-rata method and the contribution based method will produce a reduction in the value of pension benefits and because of the strong increase in the tax rate.

$\sum_{t=T+2}^{X} \Delta P_1 \cdot (1+R_1)^{T+1-t} \cdot \Omega_t$.

What happens if the rate of return on contributions decreases? Let R_2 be the new rate of return on contributions, with $R_2 < R_1$ and let ΔP_2 be the increase in pension benefits. By definition, $\sum_{t=N}^{T+1} C_t \cdot (1+R_2)^{T+1-t} \cdot \Omega_t = \sum_{t=T+2}^{X} [P + \Delta P_2] \cdot (1+R_2)^{T+1-t} \cdot \Omega_t$.

After some mathematical manipulations, we find that $\Delta P_2 = \Delta P_1 \cdot \frac{\sum_{t=T+2}^{X} \cdot (1+R_1)^{T+1-t} \cdot \Omega_t}{\sum_{t=T+2}^{X} \cdot (1+R_2)^{T+1-t} \cdot \Omega_t}$. So, $R_2 < R_1$ implies that $\Delta P_2 < \Delta P_1$. This means that if an individual decides to work one more year, the increase in the pension benefits is lower the lower the rate of return on contributions.

The increase in the retirement age at 61 after 2011 (*BERL1*), with respect to the base case, causes a sharp fall of the index for the generation born in 1950, which is the first one that must work until 61. With the second simulation (*BERL2*) there is a second strong drop for the generation born in 1955 (the first one forced to work until 63). This follows from the fact that these two generations are the first to be forced to pay more contributions and they receive a pension computed with the earning based method, so the increase in the value of their pension benefit is not much important. In contrast, the following generations are forced to pay more contributions, but receive a pension benefit computed with the pro-rata method or the contribution based method; for these generations, therefore, the increase in pension benefits is more significant and the difference between the three indexes tends to vanish. Observe, however, that the value of the index remains lower with respect to the base case in the scenarios with increased retirement age.

We can conclude that the Berlusconi reforms have a positive impact on the pension system in the medium-term but, after 2040, they appear completely ineffective: the increase in the retirement age does not induce a reduction of pension system deficits, which remain of about 1.7% of GDP. In the two next sections we evaluate different policies, an immigration policy and the reduction in pension benefits, which could be introduced in order to achieve the financial balance of the pension system in the long-run.

5 Effects of immigration policies

Can immigration policy resolve the long-run fiscal problem of population ageing? That is, given the Berlusconi reforms, how many immigrants would be necessary to balance the pension system by year 2055?

Our simulations suggest that it would be necessary to increase the yearly flow of immigrants by approximately 360,000 from 2010 onwards, i.e. to quadruple the immigration flows. Figure 17 in Appendix B shows that a more lenient immigration policy combined with the Berlusconi reform *BERL1* (indicated by *BERL1+IMM*), induces a very positive impact on the evolution of pension deficits to GDP: in 2040, pension deficits would represent 2.2% of GDP - in contrast to the 4.4% in the Berlusconi scenario - and would permit to reach the equilibrium in 2055.

The effects of this immigration policy on the demographic path are drawn in Figures 18 and 19: observe the strong reduction of the old-age dependency ratio and the increase of the share of immigrants in the total population. In particular, in 2050 the old-age dependency ratio would be 40% instead of 68% in the base case, and the (first generation) immigrants would represent 30% of the total population instead of 14.5%.[15]

From a political and sociological perspective, however, such a policy seems highly unlikely to be adopted.

We then consider the possibility of a more selective immigration policy, i.e. an immigration policy that favours "quality" rather than "quantity". From 2010 onwards all immigrants are assumed to be of a "higher quality". All immigrants are thus granted the same productivity as the natives in this scenario, and we ask the following question: How many qualified new immigrants should the country let in from year 2010 onwards to ensure the equilibrium of the pension system in 2055?

Our simulation highlights that the number of new skilled immigrants necessary to balance the pension system in 2055 is larger than the number of unskilled immigrant necessary to achieve the same objective (380,000 skilled immigrants per year *vs.* 360,000 unskilled immigrants). As shown in Figure 20, with respect to the previous immigration policy (*BERL1+IMM*), the skilled immigration policy, referred to as *BERL1+IMMSK*, provides a better evolution of the pension deficits in the medium-run.

However, in the long-run, a more selective immigration policy becomes less effective than a non-selective immigration policy. This follows from the fact that qualified immigrants that settle in Italy from 2010 onwards and retire after 2040, pay more contributions during their working life (because of higher wage rates), but also receive more important pension benefits with the application of the contribution based method. Like in the case of the increase in the retirement age, when the contribution based method is applied, the global increase in pension benefits is larger than the global increase in social contributions, so the reform becomes ineffective.

We therefore conclude that to rely immigration policies, be they selective, is not a realistic option because of the huge increase in flows of foreign labour that would be required to reach the financial equilibrium of the pension system.

[15] Second generation immigrants are treated as natives.

6 Reduction in pension benefits

If the aim of the policy maker is to reach the lung-run equilibrium of the pension system, we have already showed that neither the Berlusconi reform nor a "feasible" immigration policy are sufficient.

Increasing the contribution rate is also a useless policy, for two reasons. The first one is that pension benefits increase since, with the contribution based method introduced by the Dini reform, pension benefits are related to contributions paid. The second one is that this reform would reduce labour supply, since the reduction in net labour incomes induces individuals to substitute leisure to work.

The remaining solution is then the reduction in pension benefits.

We now simulate the Berlusconi reform (*BERL1*) combined with a reduction in the transformation coefficients applied in the computation of pensions from 2020 onwards, i.e. with the contribution based method and with the pro-rata method.[16] We find that the reduction in the transformation coefficients that is necessary to achieve the equilibrium in 2055 of the pension system is 15.9%. Figure 21 in Appendix C shows the evolution of the pension system deficits in the case of the Berlusconi reform combined with a reduction in the transformation coefficients by 15.9% - compared to the base case and to the Berlusconi reform alone -. We can see that, from 2020 onwards, pension system deficits to GDP will be lower of about 2% with respect to the base case, and that in 2055 the pension system will be in equilibrium.

The reduction in the transformation coefficients will imply a strong reduction of the replacement ratio from 2020 onwards, i.e. with the application of the pro-rata method and the contribution based method. As Table 22 shows, this policy will reduce the replacement ratio of about 11%.

The gains and losses for each generation are basically affected by two elements: the reduction in pension benefits and the evolution of taxation. As Figure 23 shows, the reform considered here (the increase in the retirement age and the reduction in pension benefits)

[16] The transformation coefficients (fixed by law) are used in the computation of the pension benefits with the contribution based method and the pro–rata method. Pension benefits are given by the capitalised value of contributions multiplied by the transformation coefficient.

has a favourable impact on the tax rate thanks to the reduction in pension deficits.

The generational accounting analysis (Figure 24) shows that this policy, as compared to the Berlusconi reform alone, implies a gain for generations born in 1950 to 1960, because they benefit the reduction of taxation without a reduction in pension benefits. For generations born between 1965 and 1980 there is a loss since, even if taxation decreases, the reduction in pension benefits is more important. For the generations born between 1990 and 2000, however, there is a gain with respect to the Berlusconi reform.

7 Conclusions

As reflected in our base scenario, the reforms introduced during the Nineties (the Amato reform in 1992 and the Dini reform in 1995) fail to ensure long-run solvability of the Italian pension system and, during the transition phase, the pension system will produce deficits as high as 3 - 5% of GDP. An additional reform is called for. In 2004 the Berlusconi government has introduced a reform that increases the retirement age after 2008.

This paper provides an evaluation of the impacts of this reform on the Italian pension system and, more generally, on the macroeconomy, by using an applied overlapping-generations general equilibrium model. We have simulated two scenarios: the first one, recently adopted by the Italian government, increases the minimum retirement age to 60 after 2008, and at age 61 after 2010. In the second scenario, the minimum retirement age could be further increased to 63 after 2015.

The results suggest that the increase in the retirement age will induce a significant improvement of the financial conditions of the pension system, but only in the short and in the medium-run. After 2040, the positive effect related to the increase in the labour supply, and then in contributions paid by the workers, is compensated by the increase in the value of pension benefits perceived by people forced to postpone retirement. The increase in the retirement age has no positive impact on the financial conditions of the pension system from 2045 onwards, and the deficit remains at about 1.7% of GDP in 2055: essentially non-affected by the Berlusconi reforms.

From the point of view of equity among the generations, the generational accounting approach shows that, with respect to the base scenario, the Berlusconi reforms will cause an important loss for the generations forced to work more, especially for the generations born in 1955 and in 1960 who receive a pension computed with the earning based method. Moreover, in the long-run, in the two simulations with the increase in the retirement age, the value of the index that measures the level of equity among the generations will be lower with respect to the base scenario.

We can conclude that the Berlusconi reform will have a very positive impact in the medium-run, but it will be completely ineffective in the long-run and will penalise the generations forced to work more.

We then explored alternative policies in order to achieve the long-run equilibrium of the pension system. Firstly, we consider an immigration policy combined with the Berlusconi reform. We showed that it is necessary to increase the number of immigrants - or the number of skilled immigrants in the case of the introduction of a selective immigration policy - by approximately 360,000 per year from 2010 onwards, i.e. to quadruple the immigration flows, in order to reach the long-run equilibrium. Since this immigration policy appears politically unfeasible, the reduction in the pension benefits remains the only solution. We showed that the reduction by 15.9% of the transformation coefficients, which corresponds to a reduction by 11% in the replacement ratios, will provide the equilibrium of the pension system in 2055.

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Appendix A

Results of the simulations related to the increase in retirement age

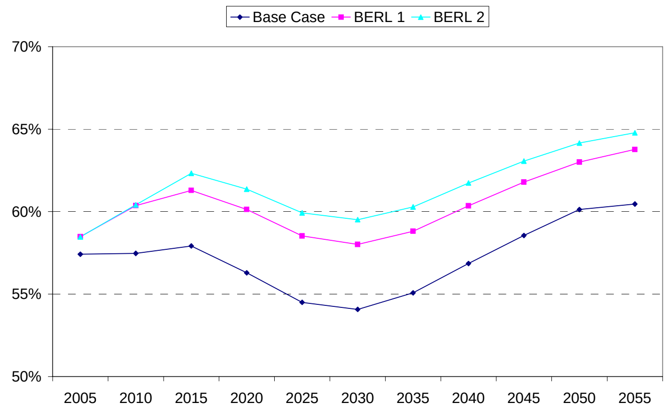

Figure 2: # workers / active population

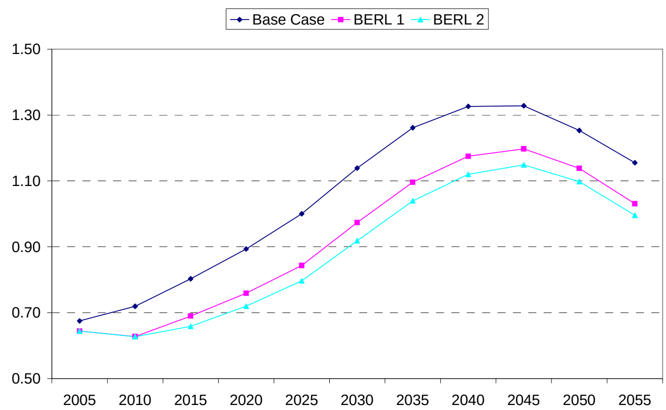

Figure 3: # retirees / # workers

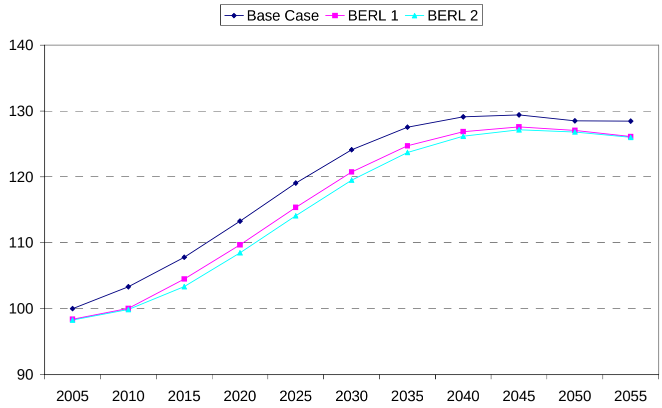

Figure 4: Wage per unit of effective labour

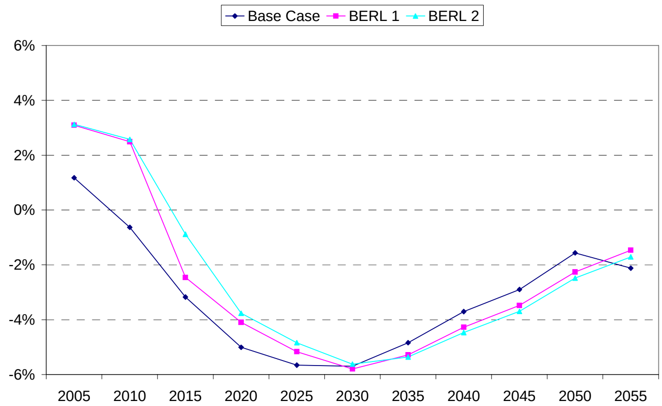

Figure 5: Rate of growth of the number of workers

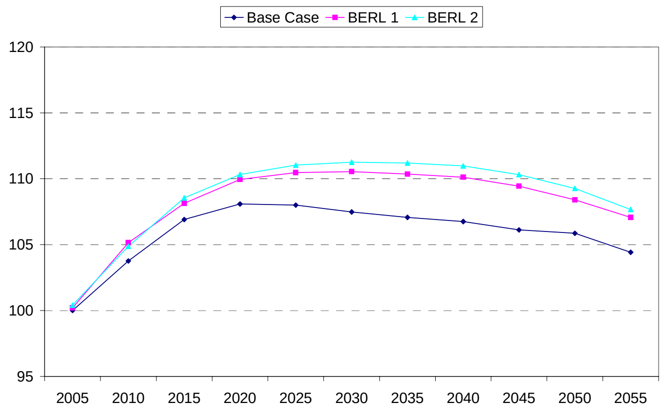

Figure 6: Time devoted to schooling

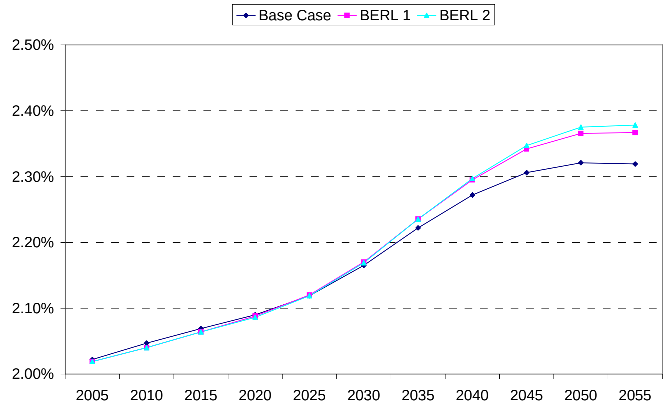

Figure 7: Productivity growth rate (g_{H_t})

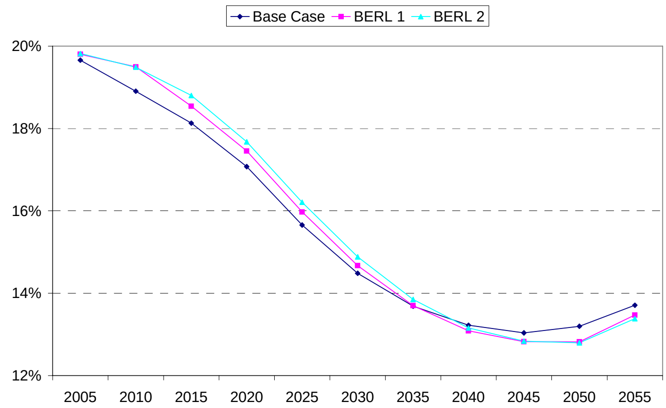

Figure 8: Investments / GDP

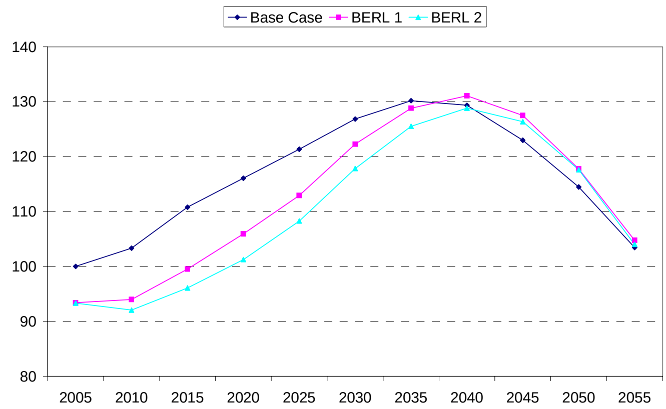

Figure 9: Tax rate

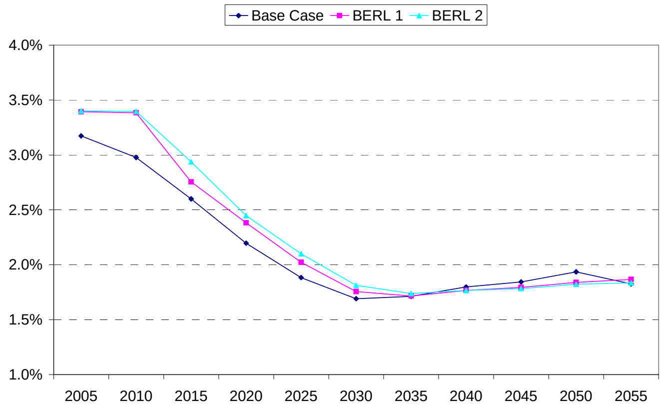

Figure 10: GDP growth rate

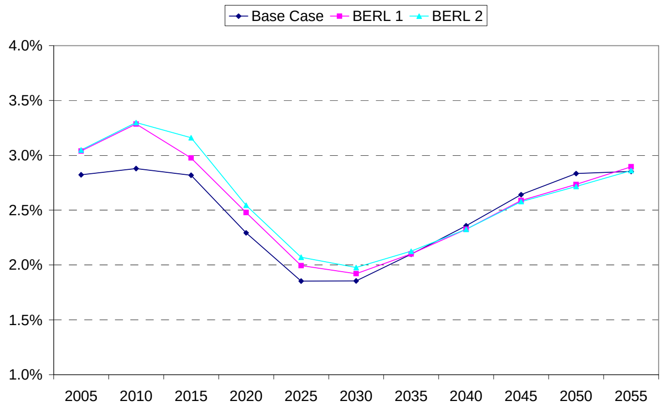

Figure 11: Per capita GDP growth rate

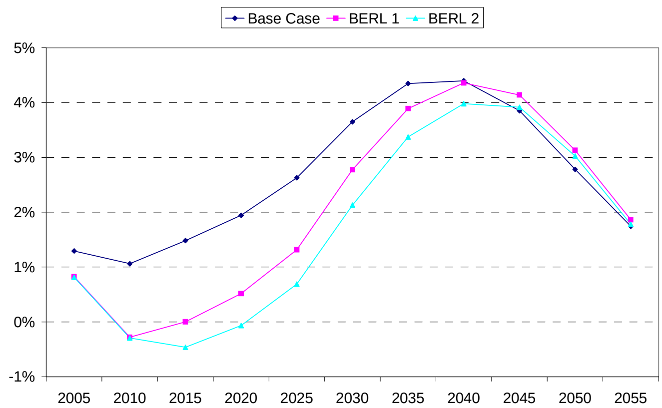

Figure 12: Pension system deficit / GDP

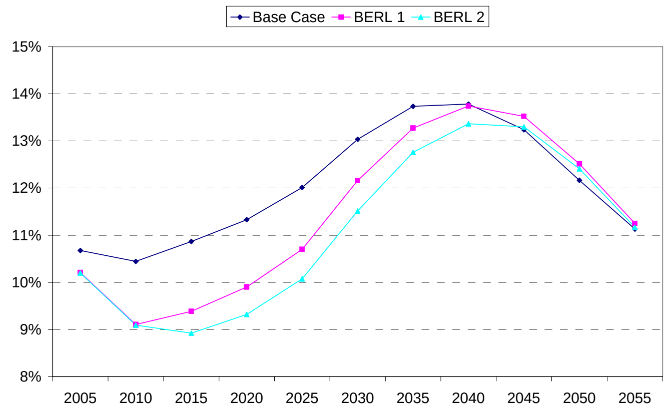

Figure 13: Pension expenditure / GDP

retirement age	57	58	59	60	61	62	63	64
years of contributions	35	36	37	38	39	40	41	42
2005	2.90%	2.70%	2.48%	2.24%	2.05%	1.85%	1.64%	1.43%
2010	2.99%	2.78%	2.56%	2.32%	2.13%	1.92%	1.71%	1.50%
2015	3.09%	2.88%	2.65%	2.42%	2.22%	2.01%	1.80%	1.59%
2020	2.67%	2.56%	2.42%	2.32%	2.21%	2.09%	1.96%	1.82%
2025	2.45%	2.37%	2.27%	2.20%	2.11%	2.02%	1.91%	1.79%
2030	2.13%	2.09%	2.03%	2.00%	1.94%	1.87%	1.79%	1.70%
2035	2.02%	1.95%	1.86%	1.83%	1.77%	1.71%	1.65%	1.57%
2040	1.92%	1.86%	1.77%	1.74%	1.69%	1.63%	1.57%	1.50%
2045	1.87%	1.81%	1.73%	1.69%	1.64%	1.59%	1.52%	1.46%
2050	1.85%	1.79%	1.71%	1.68%	1.63%	1.57%	1.51%	1.45%
2055	1.86%	1.80%	1.72%	1.69%	1.64%	1.59%	1.53%	1.46%

Table 14: Rate of return on contributions (base case)

	Base Case		*BERL1*		*BERL2*	
year of birth	*year of retiring*	*retirement age*	*year of retiring*	*retirement age*	*year of retiring*	*retirement age*
1935	1993	58	1993	58	1993	58
1940	1998	58	1998	58	1998	58
1945	2003	58	2003	58	2003	58
1950	2008	58	2011	61	2011	61
1955	2013	58	2016	61	2018	63
1960	2018	58	2021	61	2023	63
1965	2023	58	2026	61	2028	63
1970	2028	58	2031	61	2033	63
1975	2033	58	2036	61	2038	63
1980	2038	58	2041	61	2043	63
1985	2043	58	2046	61	2048	63
1990	2048	58	2051	61	2053	63

Table 15: Generations considered in the generational accounting analysis

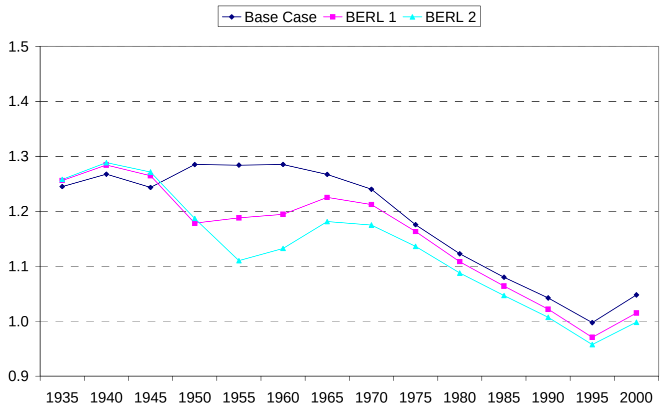

Figure 16: Expected present value of revenues / Expected present value of payments

Appendix B

Results of the simulations related to the immigration policies

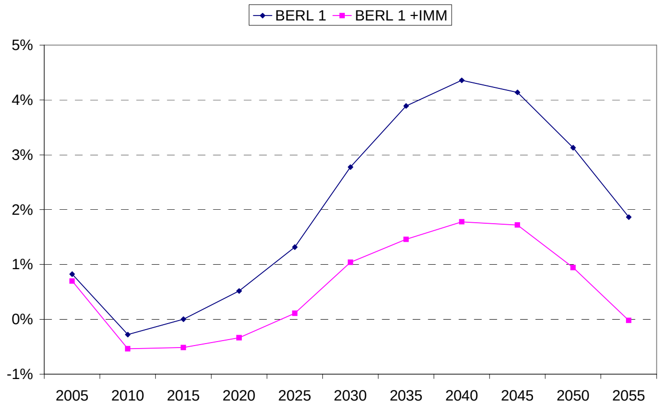

Figure 17: Pension system deficit / GDP

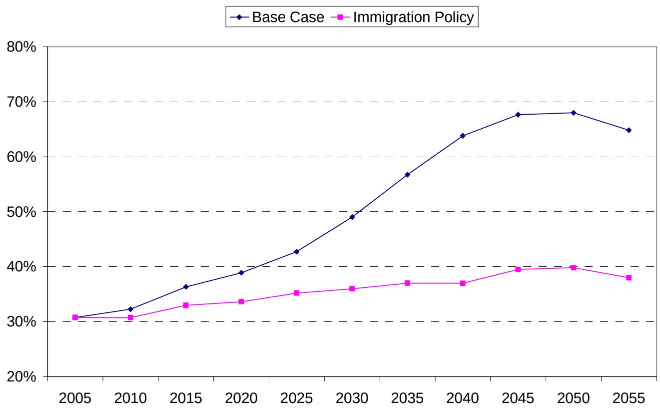

Figure 18: Old-age dependency ratio

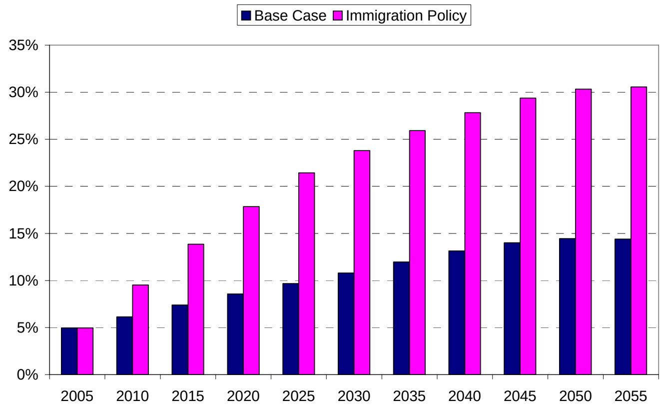

Figure 19: Immigrants / total population

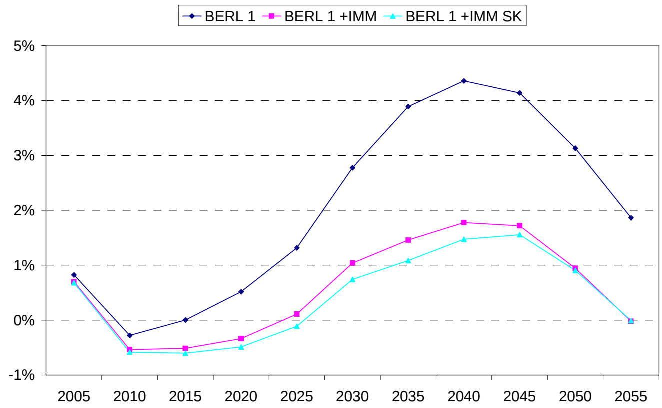

Figure 20: Pension system deficit / GDP

Appendix C

Results of the simulations related to the reduction in pension benefits

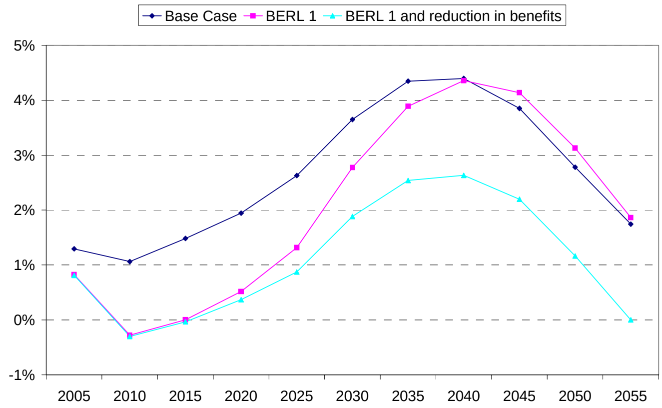

Figure 21: Pension system deficit / GDP

retirement age	62			64		
	Base Case	BERL 1	BERL 1 and reduction in benefits	Base Case	BERL 1	BERL 1 and reduction in benefits
2000	77.6%	77.6%	77.6%	81.5%	81.5%	81.5%
2010	79.4%	78.4%	78.4%	83.4%	82.3%	82.3%
2020	80.5%	82.2%	73.7%	88.6%	90.3%	81.2%
2030	73.8%	75.6%	65.1%	83.2%	85.1%	73.8%
2040	68.7%	69.3%	58.6%	78.2%	78.8%	66.7%
2050	69.4%	69.1%	58.5%	78.8%	78.5%	66.4%

Table 22: Replacement ratio for people who retire at 62 and 64

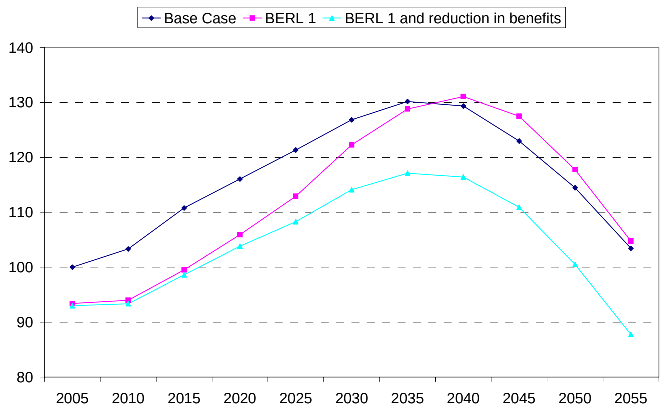

Figure 23: Tax rate

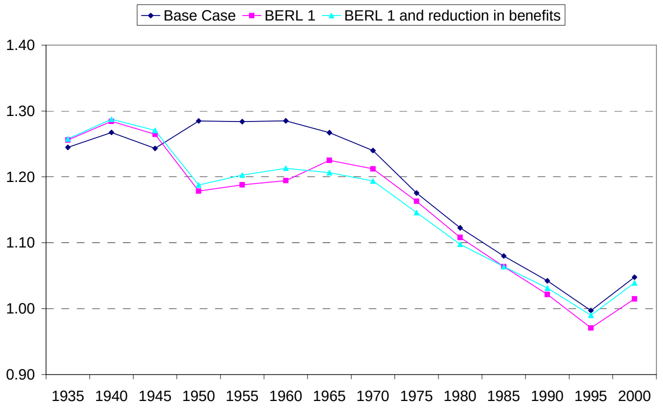

Figure 24: Present value of revenues / Present value of payments

Appendix D

Consider a reform that in 2008 increases the retirement age by one year. We want to determine, in a partial equilibrium framework, the date in which this reform becomes ineffective. Let us suppose that before the reform each individual retires at 58. The increase in the retirement age implies that individual aged 58 pay more contributions and receive an increase in pension benefits.

We note C_t the annual flow of social contributions, P_t the annual flow of pension benefits, ΔP_t the increase in pension benefits, $N_{58,2008+t}$ the number of people aged 58 that in $2008+t$ pay more contributions and by $N_{58+t,2008+t}$ the number of people aged $58+t$ that in $2008+t$ receive an increase in pension benefits. The Table below shows the gains and the losses for the pension system related to an increase of the retirement age by one year.

	gains	*losses*
2008	$N_{58,2008} \cdot (C_{2008}+P_{2008})$	0
2009	$N_{58,2009} \cdot (C_{2009}+P_{2009})$	$N_{59,2009} \cdot \Delta P_{2009}$
2010	$N_{58,2010} \cdot (C_{2010}+P_{2010})$	$N_{59,2010} \cdot \Delta P_{2010} + N_{60,2010} \cdot \Delta P_{2009}$
...	...	...
$2008+t$	$N_{58,2008+t} \cdot (C_{2008+t}+P_{2008+t})$	$N_{59,2008+t} \cdot \Delta P_{2009+t-1} + ... + N_{58+t,2008+t} \cdot \Delta P_{2009}$

We make the following assumptions:

- The number of people aged 58 grows at a constant rate n_{58}, so $N_{58,2008+t} = N_{58,2008} \cdot (1+n_{58})^t$.
- The probability that an individual survives at the end of the period, indicated by γ, is constant, so $N_{58+t,2008+t} = \gamma^t \cdot N_{58,2008}$.
- Pensions and contributions grow over time at a constant rate g, so $P_{2008+t} = (1+g)^t \cdot P_{2008}$ and $C_{2008+t} = (1+g)^t \cdot C_{2008}$.
- The rate of return on contributions is the same for each age group.

Under these assumptions, we find that:

- The number of workers in $2008+t$ that pay more contributions is:

$N_{58,2008+t} = N_{58,2008} \cdot (1+n_{58})^t$;

- The number of retirees that in $2008+t$ receive an increase in pension benefits is:

$N_{59,2008+t} + ... + N_{58+t,2008+t} =$

$\gamma \cdot N_{58,2008+t-1} + ... + \gamma^t \cdot N_{58,2008} =$

$\gamma \cdot N_{58,2008} \cdot (1+n_{58})^{t-1} + ... + \gamma^t \cdot N_{58,2008} =$

$N_{58,2008} \cdot \sum_{k=1}^{t} \left[(1+n_{58})^{t-k} \cdot \gamma^k\right]$;

- The value of more contributions perceived by the government in $2008+t$ is then:

$N_{58,2008+t} \cdot (C_{2008+t} + P_{2008+t}) = N_{58,2008} \cdot (1+n_{58})^t \cdot (1+g)^t \cdot (C_{2008} + P_{2008})$;

- Thus, the total value of the increase in pension benefits in $2008+t$ is:

$N_{59,2008+t} \cdot \Delta P_{2009+t-1} + ... + N_{58+t,2008+t} \cdot \Delta P_{2009} =$

$N_{58,2008} \cdot \Delta P_{2009} \cdot \gamma \cdot (1+n_{58})^{t-1} \cdot (1+g)^{t-1} + ... + N_{58,2008} \cdot \Delta P_{2009} \cdot \gamma^t =$

$N_{58,2008} \cdot \Delta P_{2009} \cdot \sum_{k=1}^{t} \left[(1+n_{58})^{t-k} \cdot (1+g)^{t-k} \cdot \gamma^k\right]$;

Thus, the condition for the equality in $2008+t$ between the total increase in contributions and the total increase in pension benefits is:

$$\begin{aligned} & N_{58,2008} \cdot (1+n_{58})^t \cdot (1+g)^t \cdot (C_{2008} + P_{2008}) \\ = \;& N_{58,2008} \cdot \Delta P_{2009} \cdot \sum_{k=1}^{t} \left[(1+n_{58})^{t-k} \cdot (1+g)^{t-k} \cdot \gamma^k\right] \end{aligned}$$

that can be rewritten:

$$C_{2008} + P_{2008} = \Delta P_{2009} \cdot \sum_{k=1}^{t} \left[\frac{(1+n_{58}) \cdot (1+g)}{\gamma}\right]^{-k}$$

Given the hypothesis that the rate of return on contributions is the same for each age group, the increase in pension benefits is $\Delta P_{2009} = \frac{C_{2008}+P_{2008}}{\sum_{k=59}^{95} (1+R)^{58-k} \cdot \frac{\Omega_k}{\Omega_{58}}}$ (see note 21), where $\frac{\Omega_k}{\Omega_{58}}$ represents the probability that an individual, who is alive at age 58, will be alive at age k, so that $\frac{\Omega_k}{\Omega_{58}} = \gamma^{k-58}$.

Then, we have:

$$\widetilde{R} = \sum_{k=1}^{t} \eta^{-k}$$

where $\widetilde{R} = \sum_{k=59}^{95} \left(\frac{1+R}{\gamma}\right)^{58-k}$ and $\eta = \frac{(1+n_{58})\cdot(1+g)}{\gamma}$.

Then,

$$\widetilde{R} = \frac{1-\eta^{-t}}{\eta - 1}$$

Hence, the date t at which gains and losses equalise is the following:

$$t = -\frac{\log\left[1-(\eta-1)\cdot\widetilde{R}\right]}{\log\eta}$$

Given these hypothesis indicated above and for γ=97%, g=3%, n_{58}=-1.2% and a rate of return on contributions equal to 2.3%, we find that t=30. In a partial equilibrium analysis, a reform that increases the retirement age by one year stops being effective at year 2038.

On introducing heterogeneous discrete-choice agents in applied GE models

Abstract

Our paper contributes to bridge the gap between the micro-simulation's approach and applied GE models, by making use of exact aggregation results from the discrete choice literature: heterogeneous individuals choosing continuous amounts within a set of discrete alternatives may be aggregated into a representative agent with CES/CET preferences. These results therefore provide a natural link between the two policy evaluation approaches. We illustrate the usefulness of these results by evaluating potential effects of population ageing on the dynamics of income distribution and inequalities, using a simple OLG model when individuals have to make leisure/work decisions, and choose a profession among a discrete set of alternatives.

This paper is joint with Jean Mercenier, THEMA - Université de Cergy-Pontoise, 33 Bd du Port, 95011 Cergy-Pontoise Cedex.

1 Introduction

During the last twenty years, applied GE models have become standard tools of quantitative policy assessment. Their appeal has built on their rigorous grounding in economic theory: individual agents' decision-making behaviour are derived from explicit optimisation under strictly specified technological or budget constraints, given market signals that ensure global consistency. These theoretical foundations have made applied GE models appear particularly useful for ex-ante evaluations of policy reforms. Convincing as this argument may be, it can only be sustained if ex-post performance evaluations are made, and sources of prediction errors identified and taken care of. Amazingly, the methodology has rarely been submitted to such tests: notable exceptions are Kehoe *et al.* (1995) and Kehoe (2003). Though the former's conclusion – based on a single-country perfectly-competitive model of the 70s' – sounded rather positive and optimistic, the latter's assessment – built on three of the most prominent applied GE models constructed to predict the impact of NAFTA – is quite devastating: "Theses models drastically underestimated the impact of NAFTA on North American trade. Furthermore, the models failed to capture much of the relative impacts on different sectors." (Kehoe, 2003, p0).

Many reasons could of course contribute to explain why most applied GE models would probably fail to a serious ex-post prediction test. The theoretical mechanism hypothesised in the model may not be appropriate: Kehoe (2003), for example, suggests that "no plausible parameter changes can get the models of NAFTA built on the Dixit-Stiglitz specification to match what actually has happened in North America". Another reason, often (over- ?) stressed by statistically oriented econometricians, is that applied GE modellers tend to excessively rely on guesstimated rather than on rigorously estimated parameter values; more generally, that applied GE modellers pay too little attention on the data-set they use (see *e.g.* Mercenier and Yeldan, 1999). Yet another – and potentially more serious – reason is that the whole apparatus relies on the concept of "representative agent" despite unclear aggregation procedures to link these aggregate optimising decision-makers to the numerous individual agents whose behaviour they are meant to capture.

During the same period, microsimulation models have also become increasingly popular tools for policy analysis precisely because they avoid any reliance on *typical agents* by fully taking into account the heterogeneity of individuals as they are observed in micro-data sets. See Bourguignon and Spadaro (2005), for an excellent survey, and Hagneré *et al.* (2003) for an application to the French labour market. Indeed, working with myriads of actual economic agents rather than with a few hypothetical ones makes it possible to precisely identify the winners and the losers of a reform – obviously a major concern to policy-makers – yet, making it possible by simple addition to accurately measure this impact on aggregate variables. The increasing availability of large and detailed data-sets on individual agents makes this quite appealing. The drawback of this approach is of course that it is partial equilibrium in essence: individuals' labour supply adjustments to some new tax incentive scheme may be accurately captured for given wages and other policy parameters, but market equilibrium and government budget constraints can be expected to have a feedback influence that is typically neglected. One could of course imagine iterations between the microsimulation and the applied GE models, and indeed, a few efforts have successfully been done in this direction: see for instance Savard (2003) and Arntz *et al.* (2006). Though this iterative strategy might indeed be satisfactory for some problems – in particular when dynamics are thought unimportant – it is likely to be unfeasible for those requiring more sophisticated apparatus such as OLG models. Analysing policy issues in a context of changing demography, for instance, would obviously require a different approach. It is the object of this paper to suggest one such approach.

Our paper contributes to bridge the gap between the two approaches by making use of some simple yet powerful exact aggregation results due to Anderson, de Palma and Thisse (1992) (here after: AdPT). Indeed, they show that, under reasonably mild conditions, heterogeneous individuals that have to choose continuous amounts within a set of (possibly subsets of) discrete alternatives may be aggregated into a representative agent with (possibly multiple-level) CES preferences. These results therefore provide a natural and appealing link between the standard applied GE apparatus and the microsimulations approach. It also makes readily available to applied GE modellers a growing body of empirical results drawn from panel-data econometrics. There is no free lunch, unfortunately: some details captured by the microsim approach could be lost in the aggregation, a cost that one should balance against the benefits of accounting for the GE feedbacks.

We illustrate the usefulness of these results in the context of a simple OLG model.

Simulations will be done in vitro – i.e., using a computer generated data-set – to explore the potential consequences of population ageing on the dynamics of income distribution and inequalities, when individuals have to make leisure/work decisions, and choose a profession among a discrete set of alternatives.

The paper is organised as follows: in Section 2, we provide a refresher on probabilistic discrete choice models, and show how the basic aggregation results emerge from assuming multinomial logit heterogeneity in preferences. We then apply these results in Section 3 to modelling nested choices between leisure/work - professions and imbed this decision problem into an OLG model that is sketched in Section 4. We then submit in Section 5 the economy to an ageing shock, and plug into individual decision problems the computed equilibrium prices to evaluate the effect of population ageing on the dynamic path of income inequality indicators. The paper closes with a brief conclusion.

2 Discrete-choice models

2.1 A refresher

Assume a population of individuals $h = 1, ..., N$ has to choose among a set $i, j = 1, ..., n$ of discrete alternatives with associated utility levels:

$$\widetilde{u}_j^h = u_j + \epsilon_j^h \qquad j = 1, ..., n$$

where u_j is a deterministic component (for now, assumed common to all individuals) and ϵ_j^h is a random term. Each h is therefore characterised by a draw $\epsilon = (\epsilon_1^h, ..., \epsilon_n^h)$ in a probability distribution with cumulative density function $F(\epsilon)$. Assume that individuals in this population are not only statistically identical but also statistically independent. Then, the distribution of choices is multinomial with mean $\overline{X}_j = NP_j$, $j = 1, ..., n$, where P_j denotes the probability that alternative j be chosen by h. $\overline{X}_j$ is the math expectation of demand for alternative j; for N large enough, $\overline{X}_j$ is a close approximation of aggregate demand for j in this population. In other words, aggregate demands for each alternative may be readily determined from the individual discrete choice problem.

The probability that h will choose alternative j is:

$$\begin{aligned} P_j &= prob\left[\widetilde{u}_j^h \geq \widetilde{u}_k^h, \forall k = 1, ..., n\right] \\ &= prob\left[u_j + \epsilon_j^h \geq u_k + \epsilon_k^h, \forall k = 1, ..., n\right] \\ &= prob\left[\epsilon_k^h - \epsilon_j^h \leq u_j - u_k, \forall k = 1, ..., n\right] \end{aligned}$$

The determination of the choice probabilities using $F(\epsilon)$ is in principle always possible but in general extremely difficult, in particular if ϵ is assumed normally distributed, as would seem natural.[1] Fortunately, a theorem due to Mc Fadden identifies a class of distribution functions $F(\epsilon)$ – of which the multinomial logit is a special case – for which these probabilities may be easily determined indirectly. Consider the generalised extreme value distribution function

$$F(\epsilon_1, ..., \epsilon_n) = \exp\left[-H(e^{-\epsilon_1}, ..., e^{-\epsilon_n})\right]$$

with H a nonnegative function defined over R_+^N satisfying the following properties: (i) H is homogeneous of degree $1/\mu$; (ii) $lim_{x_i \to \infty} H(x_1, ..., x_n) = \infty \; \forall i = 1, ..., n$; (iii) the mixed partial derivatives of H with respect to k different variables exist and are continuous, non-negative if k is odd, non-positive if k is even, $k = 1, ..., n$.[2] Then, the choice probabilities P_j may be determined as:

$$P_j = \mu \frac{\partial \ln H(e^{u_1}, ..., e^{u_n})}{\partial u_j}$$

It can easily be checked that the following particularisation of H,

$$H(\epsilon_1, ..., \epsilon_n) = \sum_{j=1}^{n} \epsilon_j^{1/\mu}$$

satisfies the previous properties. The cumulative distribution function becomes:

$$F(\epsilon_1, ..., \epsilon_n) = \exp\left[-\sum_{j}^{n} e^{-\epsilon_j/\mu}\right] = \prod_{j}^{n} \exp\left[-e^{-\epsilon_j/\mu}\right]$$

[1] In that case – the well known probit – there is no explicit form for the choice probabilities: to determine P_j requires solving numerically $(n-1)$-dimensional integrals.

[2] These technical conditions are needed to ensure that $F(\epsilon)$ is indeed a cumulative probability distribution function.

that is, the product of n i.i.d. double exponential distributions characterises the stochastic behaviour of utilities $\widetilde{u}_j$, and it follows from the theorem that

$$P_j = \mu \frac{\partial \ln \sum_i^n e^{u_i/\mu}}{\partial u_j} = \frac{e^{u_j/\mu}}{\sum_i^n e^{u_i/\mu}} \tag{1}$$

which are the choice probabilities derived from a multinomial-logit population with dispersion parameter μ. This of course makes the MNL quite appealing. It turns out that, in addition, it provides a good approximation to the normal distribution.[3] Observe that, from (1),

$$\frac{\partial P_j}{\partial u_k} = -\frac{P_j P_k}{\mu} \qquad j,k = 1,...n,\ j \neq k$$

so that the cross-elasticities

$$Elas(P_j, u_k) = -\frac{P_k u_k}{\mu} \qquad j,k = 1,...n,\ j \neq k$$

are independent of j. Any change in the deterministic utility level associated with alternative k will therefore affect symmetrically the choice probabilities of all other alternatives: relative aggregate demands between two alternatives are unaffected by variations in the utility level of a third alternative. This over-restrictive property, known as the *independence of irrelevant alternatives* can be bypassed by nesting multinomial logit systems, as we shall now illustrate.

Assume that the set A of alternatives $j = 1,...,n$ can be partitioned into m subsets $\{A_l; l = 1,...,m\}$ of close alternatives. We particularise the $H(\epsilon_1,...,\epsilon_n)$ function as follows:

$$H_A(\epsilon_1,...,\epsilon_n) = \sum_{l=1}^{m} \left[\sum_{i \in A_l} \epsilon_i^{1/\mu_2} \right]^{\mu_2/\mu_1} \tag{2}$$

This function is homogeneous of degree $1/\mu_1$; Mc Fadden has shown that if $\mu_1 \geq \mu_2$, this function satisfies all the properties required to apply the extreme value theorem. It follows that

$$F(\epsilon_1,...,\epsilon_n) = \exp\left\{ -\sum_{l}^{m} \left[\sum_{i \in A_l} e^{-\epsilon_i/\mu_2} \right]^{\mu_2/\mu_1} \right\}$$

[3] Ben Akiva and Lerman (1985, p128) write: "there is still no evidence to suggest in which situations the greater generality of the multinomial probit is worth the additional computational problems resulting from its use." We are not aware that such evidence has been reported in the literature since then.

and

$$
\begin{aligned}
P_j &= \mu_1 \frac{\partial \ln \sum_l^m \left[\sum_{i \in A_l} e^{u_i/\mu_2}\right]^{\mu_2/\mu_1}}{\partial u_j} \\
&= \frac{\left[\sum_{i \in A_l} e^{u_i/\mu_2}\right]^{\mu_2/\mu_1}}{\sum_l^m \left[\sum_{i \in A_l} e^{u_i/\mu_2}\right]^{\mu_2/\mu_1}} \cdot \frac{e^{u_j/\mu_2}}{\sum_{i \in A_l} e^{u_i/\mu_2}} \qquad j \in A_l \qquad (3)
\end{aligned}
$$

This expression has a structure that makes it straightforward to understand. The second term is the probability that, within the subset A_l of alternatives, j be chosen. The first term represents the probability that among all subsets of A, A_l be chosen.

The expression can be given an alternative welfare interpretation. To see this, consider a subset A_l of alternatives, and define

$$
\begin{aligned}
H_{A_l} &= H_{A_l}(\epsilon_i, i \in A_l) = \sum_{i \in A_l} \epsilon_i^{1/\mu_2} \qquad (4) \\
G_{A_l} &= G_{A_l}(u_i, i \in A_l) = \mu_2 \ln \sum_{i \in A_l} e^{u_i/\mu_2} \qquad (5)
\end{aligned}
$$

It can be shown (see AdPT, p60) that G_{A_l} is the *expected value of the maximum of utilities from the alternatives in subset* A_l , which can therefore be interpreted as a measure of the attractiveness of the subset A_l. Dividing G_{A_l} by μ_1 and using an exponential transform yields:

$$
e^{G_{A_l}/\mu_1} = \left[\sum_{i \in A_l} e^{u_i/\mu_2}\right]^{\mu_2/\mu_1}
$$

Upon substitution of H_{A_l} into (2), we get:

$$
H_A = H_A(H_{A_l}, l = 1, ..., m) = \sum_l^m [H_{A_l}]^{\mu_2/\mu_1}
$$

Note the similarity of this expression with (4). We can write the expected value of the maximum of utilities from choosing between the different subsets of alternatives as:

$$
G_A = \mu_1 \ln \sum_l^m e^{G_{A_l}/\mu_1}
$$

which can be transformed to yield:

$$
e^{G_A/\mu_1} = \sum_l^m e^{G_{A_l}/\mu_1} = \sum_l^m \left[\sum_{i \in A_l} e^{u_i/\mu_2}\right]^{\mu_2/\mu_1}
$$

Hence, making use of those expressions into (3), the probability P_j takes a intuitive structure:

$$P_j = \frac{e^{G_{A_l}/\mu_1}}{\sum_l^m e^{G_{A_l}/\mu_1}} \cdot \frac{e^{u_j/\mu_2}}{\sum_{i \in A_l} e^{u_i/\mu_2}} \tag{6}$$

Comparing (6) with (3), we see that the first term is a logit choice probability between $l = 1, ..., m$ alternatives, each alternative being priced by the expected maximum utilities from alternatives belonging to subset A_l. The nested discrete choice problem can therefore quite simply be solved sequentially, one level after the other, up the decision tree. It is immediate to generalise this to any number q of nested discrete choices, provided that $\mu_1 \geq \mu_2 \geq ... \geq \mu_q$ where q is the lowest level in the decision tree, i.e. where individual heterogeneity is lowest.

2.2 From individual discrete choices to collective CES preferences

For many interesting problems, the individual not only has to decide *which* among many possible alternatives, but also *how much* given the previous choice. Also, the non idiosyncratic part of the utilities will in general depend on some exogenous characteristics of both the options, such as market prices, and the decider, such as age, sex etc. We now generalise.

Let subscript 0 refer to an option outside the set of discrete alternatives, acting as the *numeraire*, and write:

$$u_j = \ln x_j + \alpha \ln x_0 \qquad j = 1, ..., n$$

where x_j is a real variable measuring the quantity chosen conditional on option j being selected. Hence, from the budget constraint, $x_0 = y - p_j x_j$ where y is income. Substituting this into the preferences, and maximising u_j with respect to x_j yields the conditional demand function:

$$x_j = \frac{y}{(1 + \alpha) \cdot p_j} \tag{7}$$

so that the indirect utility has the form:

$$\widetilde{v}_j = const + (1 + \alpha) \ln y - \ln p_j + \epsilon_j \qquad j = 1, ..., n$$

where *const* depends only on α. Note that adding some individual exogenous characteristics z_j into this equation only affects the valuation of the option by the individual:

$$\begin{aligned}\widetilde{v}_j &= const + (1+\alpha)\ln y - \ln p_j + \beta_j z_j + \epsilon_j \\ &= const + (1+\alpha)\ln y - \ln\left(p_j / f(z_j)\right) + \epsilon_j\end{aligned}$$

where $f(z_j) = \exp(\beta_j z_j)$.

Assuming the random terms $\epsilon_1, ..., \epsilon_n$ are i.i.d. double exponentials, we know that the choice probabilities are given by the MNL as:

$$P_j = \frac{\exp\left(v_j/\mu\right)}{\sum_i^n \exp\left(v_i/\mu\right)} \qquad j = 1, ..., n$$

where v_j is the deterministic part of the conditional indirect utility associated with alternative j; that is:

$$P_j = \frac{\exp\left\{\left(-\ln p_j + K\right)/\mu\right\}}{\sum_i^n \exp\left\{\left(-\ln p_i + K\right)/\mu\right\}} \qquad j = 1, ..., n$$

where all the terms independent of i, j have been collected into $K = (1+\alpha)\ln y + \alpha \ln \alpha - (1+\alpha)\ln(1+\alpha)$; these terms cancel out so that:

$$\begin{aligned}P_j &= \frac{\exp\left\{-\ln p_j/\mu\right\}}{\sum_i^n \exp\left\{-\ln p_i/\mu\right\}} \qquad j = 1, ..., n \\ &= \frac{p_j^{-1/\mu}}{\sum_i^n p_i^{-1/\mu}} \end{aligned} \tag{8}$$

Since individual conditional demands are given by (7), the expected aggregate demand (aggregated over the N individuals assuming all have the same income y) becomes:

$$\begin{aligned}X_j &= N \cdot \frac{y}{(1+\alpha)\cdot p_j} \cdot P_j = \frac{P_j}{p_j} \cdot \widehat{Y} \qquad j = 1, ..., n \\ X_0 &= N \cdot \alpha \cdot \frac{y}{(1+\alpha)} = \alpha \widehat{Y}\end{aligned}$$

where $\widehat{Y} = \frac{Y}{1+\alpha} = \frac{Ny}{1+\alpha}$ is the aggregate income spent on the set of discrete options. Replacing the P_j by their expression from (8), we finally get that:

$$\begin{aligned}X_j &= \frac{p_j^{-(1+\mu)/\mu}}{\sum_i^n p_i^{-1/\mu}} \cdot \widehat{Y} \qquad j = 1, ..., n \\ X_0 &= \alpha \widehat{Y}\end{aligned} \tag{9}$$

It is easy to show that these aggregate demands are exactly those obtained from the representative agent choosing from the following preferences:

$$U = \left[\sum_{i=1}^{n} X_i^{\rho}\right]^{1/\rho} \cdot X_0^{\alpha}$$

with $0 \leq \rho \leq 1$ and $\alpha > 0$. Indeed, maximising U subject to the aggregate budget constraint $Y = \sum_i p_i X_i + X_0$, we get

$$\begin{aligned} X_j &= \frac{p_j^{-1/(1-\rho)}}{\sum_i^n p_i^{-\rho/(1-\rho)}} \cdot \widehat{Y} \qquad i = 1, ..., n \\ X_0 &= \alpha \widehat{Y} \end{aligned} \tag{10}$$

Setting $\mu = (1-\rho)/\rho$ and comparing (10) with (9) proves the exact aggregate result established by AdPT.

3 Modelling leisure/work decisions and the choice of a profession

3.1 Discrete choice formulation

Consider an individual h with given characteristics such as sex, age-class etc. belonging to a population *with the same socio-economic characteristics*.[4] He has to decide whether to work or not, and if he does, in which profession. We model this as a two-level discrete choice problem; we take advantage of the nested structure to solve the problem sequentially starting with the choice of profession.

3.1.1 Choosing between professions

There are I possible professions indexed i, j. We write the utility as a log-linear function:

$$\widetilde{v}_i^h = \ln \theta_i + \ln w_i + \epsilon_i^h \qquad i = 1, ..., I$$

[4] The population can be partitioned into k=1,...,K cells according to as many characteristics as made possible by the available data. In what follows, we model the decision problems of individuals belonging to one such cell, and neglect the subscript of the cell to ease notation. In the applied GE model there will be one representative agent for each cell.

The first term captures the (common to all options) disutility of working as well as the welfare costs/benefits of various characteristics specific to the profession, and w_i is the market wage (adjusted for characteristic-specific efficiency) expressed in terms of the consumption good. Note that these two terms are common to all h within the considered population cell.[5] Intra-cell individual heterogeneity in preferences is captured by the i.i.d. double exponential stochastic term ϵ_i^h with dispersion parameter μ.[6] From the previous section, we know that the probability that h will choose profession i is:

$$\begin{aligned} P_i &= \frac{\exp\left(\frac{\ln\theta_i+\ln w_i}{\mu}\right)}{\sum_j \exp\left(\frac{\ln\theta_j+\ln w_j}{\mu}\right)} \\ &= \frac{\theta_i^{1/\mu}\cdot w_i^{1/\mu}}{\sum_j \theta_j^{1/\mu}\cdot w_j^{1/\mu}} \end{aligned}$$

3.1.2 Choosing whether to work or not

Let the utility h enjoys from not working be:

$$\widetilde{V}_0^h = \ln\Theta_0 + \varepsilon_0^h$$

where ε_0^h is a random term which captures individual heterogeneity in the valuation of leisure (the disutility of working). The alternative is for the individual to work, taking into account that if he does so, he will be able to choose the best profession. The valuation of the alternative *work* that is consistent with the second stage decision problem is:

$$\widetilde{V}_1^h = V_1 + \varepsilon_1^h$$

[5] We therefore assume here that, upon making their optimal decisions, individuals ignore possible within-cell idiosyncratic productivity differences, that will ex-post be responsible for the observed distribution of wages in the data. The additional information contained in the within-cell distribution of individual wages w_i^h will be used in the calibration of the general equilibrium model, in the econometric estimation of the parameters of the discrete-choice preferences, and in the ex-post microsimulations. We will refer to this within-cell average wage w_i as the Mincer-wage for that cell.

[6] The mean of a double exponential distribution is given by $\mu\cdot\ \gamma$, where γ is the Euler's constant (≈ 0.577), and the variance is given by $\frac{\pi^2\mu^2}{6}$ (see e.g. AdPT p58-60).

where:

$$
\begin{aligned}
V_1 &= \mu \ln \sum_i \exp\left(\frac{\ln \theta_i + \ln w_i}{\mu}\right) \\
&= \mu \ln \sum_i \theta_i^{1/\mu} \cdot w_i^{1/\mu}
\end{aligned}
$$

We assume that $\varepsilon_0^h, \varepsilon_1^h$ are double exponential i.i.d. random terms with dispersion parameter υ. The probability that h will choose to *farniente* is therefore:

$$
\begin{aligned}
P_0 &= \frac{\Theta_0^{1/\upsilon}}{\Theta_0^{1/\upsilon} + \exp(V_1/\upsilon)} \\
&= \frac{\Theta_0^{1/\upsilon}}{\Theta_0^{1/\upsilon} + \left[\sum_i \theta_i^{1/\mu} \cdot w_i^{1/\mu}\right]^{\mu/\upsilon}}
\end{aligned}
$$

3.1.3 Aggregation of individual choices

Let there be N individuals in this population cell, N large. The within-cell aggregate labour supply of each profession resulting from individual discrete choices is then easily established:

$$
\begin{aligned}
L &= (1 - P_0) \cdot N \\
&= \frac{\left[\sum_i \theta_i^{1/\mu} \cdot w_i^{1/\mu}\right]^{\mu/\upsilon}}{\Theta_0^{1/\upsilon} + \left[\sum_i \theta_i^{1/\mu} \cdot w_i^{1/\mu}\right]^{\mu/\upsilon}} \cdot N
\end{aligned}
$$

and therefore:

$$
\begin{aligned}
L_i &= P_i L \qquad (11) \\
&= \frac{\theta_i^{1/\mu} \cdot w_i^{1/\mu}}{\sum_j \theta_j^{1/\mu} \cdot w_j^{1/\mu}} \cdot \frac{\left[\sum_j \theta_j^{1/\mu} \cdot w_j^{1/\mu}\right]^{\mu/\upsilon}}{\Theta_0^{1/\upsilon} + \left[\sum_j \theta_j^{1/\mu} \cdot w_j^{1/\mu}\right]^{\mu/\upsilon}} \cdot N \qquad i = 1, \cdots, I
\end{aligned}
$$

3.2 The representative agent formulation

Our next task is to write an optimisation problem for a representative agent[7] seeking to split his total time N between leisure and professional activities, such that the optimal allocation coincides with the one generated from aggregation of individual discrete choices (11). We proceed in two steps.

[7] One for each population-cell, but here again we neglect the cell index k to ease notation.

We first determine the optimal share of total time N between leisure and work. Let $S_{\mathcal{L}}$ and S_L denote the amount of time devoted to leisuring and working, and λ be the household's relative valuation of leisure. (The index λ is of course inversely related to market wages in a way that will be established later, but is here assumed given.) He chooses $S_{\mathcal{L}}$ and S_L so as to maximise $\lambda S_{\mathcal{L}} + S_L$ subject to a CET constraint

$$\left(\alpha_{\mathcal{L}} \left[S_{\mathcal{L}}\right]^{\frac{\tau+1}{\tau}} + \alpha_L \left[S_L\right]^{\frac{\tau+1}{\tau}}\right)^{\frac{\tau}{\tau+1}} = 1$$

that captures the fact that moving in and out of the job market is not costless. It immediately follows from the FOC that the optimal ratio of time spent on the two activities is:

$$\frac{S_{\mathcal{L}}}{S_L} = \left[\frac{\alpha_{\mathcal{L}}}{\alpha_L}\right]^{-\tau} \cdot \lambda^{\tau} \tag{12}$$

Making use of (12) jointly with the resource constraint $\mathcal{L}+L = N$ yields the household's optimal labour supply:

$$L = \frac{\alpha_L^{-\tau}\lambda^{-\tau}}{\alpha_{\mathcal{L}}^{-\tau} + \alpha_L^{-\tau}\lambda^{-\tau}} \cdot N \tag{13}$$

The second step of the decision problem consists to allocate this work time between professions taking into account relative market wages and the mobility cost between professions. Formally, the problem is the following:

$$\begin{cases} \underset{s_i}{\text{Max}} & \sum_i w_i s_i \\ \text{s.t.} & \left[\sum_i \alpha_i \cdot s_i^{\frac{\sigma+1}{\sigma}}\right]^{\frac{\sigma}{\sigma+1}} = 1 \end{cases}$$

This yields the optimal shares:

$$\frac{s_i}{s_j} = \left[\frac{\alpha_i}{\alpha_j}\right]^{-\sigma} \cdot \left[\frac{w_i}{w_j}\right]^{\sigma} \qquad i \neq j$$

which, jointly with the resource constraint $\sum_i L_i = L$ determines the amount of time devoted to working in each profession:

$$L_i = \frac{\alpha_i^{-\sigma} \cdot w_i^{\sigma}}{\sum_j \alpha_j^{-\sigma} \cdot w_j^{\sigma}} \cdot L$$

Making use of (13), we can substitute out L and get:

$$L_i = \frac{\alpha_i^{-\sigma} \cdot w_i^{\sigma}}{\sum_j \alpha_j^{-\sigma} \cdot w_j^{\sigma}} \cdot \frac{\alpha_L^{-\tau} \cdot \lambda^{-\tau}}{\alpha_{\mathcal{L}}^{-\tau} + \alpha_L^{-\tau} \cdot \lambda^{-\tau}} \cdot N \tag{14}$$

Let the household's relative leisure valuation index λ be inversely related to market wages by the following function:

$$\lambda = \alpha_L^{-1} \left[\sum_j \alpha_j^{-\sigma} \cdot w_j^{\sigma} \right]^{-\frac{1}{\sigma}}$$

so that (14) can be rewritten as:

$$L_i = \frac{\alpha_i^{-\sigma} \cdot w_i^{\sigma}}{\sum_j \alpha_j^{-\sigma} \cdot w_j^{\sigma}} \cdot \frac{\left[\sum_j \alpha_j^{-\sigma} \cdot w_j^{\sigma}\right]^{\frac{\tau}{\sigma}}}{\alpha_{\mathcal{L}}^{-\tau} + \left[\sum_j \alpha_j^{-\sigma} \cdot w_j^{\sigma}\right]^{\frac{\tau}{\sigma}}} \cdot N \tag{15}$$

Comparing this expression with (11), we see that, though the interpretation of the parameters differ considerably, the two expressions are identical provided that we set:

$$\begin{cases} \sigma = 1/\mu \\ \tau = 1/\upsilon \\ \alpha_i = 1/\theta_i \\ \alpha_{\mathcal{L}} = 1/\Theta_0 \end{cases} \tag{16}$$

For each population cell, there are N^k, $k = 1, ..., K$, individuals facing a specific Mincer-wage vector w_i^k, $i = 1, ..., n$, and having preference characteristics θ_i^k, $\mu^k, \Theta_0^k, \upsilon^k$. These parameters can be estimated using discrete choice econometric techniques (see e.g. Train, 2003, for a recent thorough exposition), and the aggregate labour-supply systems (15) plugged into the general equilibrium model.[8]

3.3 The OLG set-up

We now have, for each population cell, a different labour-supply system generated from aggregation of individual discrete choices, that is, there are as many representative labour-supplying agents as there are socioeconomic characteristics of interest in the micro database. This could suggest that, without restrictions on the number of these characteristics, we would rapidly run into the "curse of dimensionality" in the general equilibrium set-up, which would of course drastically limit the appeal of the current approach. Fortunately,

[8] There are some econometric issues here, such as the correction of the selectivity bias when estimating an equation over an endogenously selected population (see Bourguignon *et al.* (2004), for a discussion in a multinomial logit context) but these issues are beyond the scope of this paper.

this is not the case. Indeed, if we adopt identical and standard homothetic intertemporal preferences, we can aggregate further these representative labour-supplying agents into a single (per-generation) representative consumer that optimally allocates its human wealth to lifetime consumption.

We distinguish between G generations that coexist at each time period t. At the end of each period, the oldest group $g(G)$ disappears and a new generation $g(1)$ enters the active population according to the following rule:

$$N_{g(1),t+1} = \eta_t \cdot N_{g(1),t} \tag{17}$$

where $N_{g(1),t}$ denotes the number of young people at time t and η_t is an exogenous gross reproduction rate. Each agent maximises its intertemporal utility subject to its wealth constraint. Doing so, he chooses: *(a)* the intertemporal profile of consumption (and therefore of assets accumulation); *(b)* how much to work, and in which profession (for those generations that are active, retirement is exogenously fixed at some late age); *(c)* the amount of bequests to leave when exiting. Formally, lifetime utility for the generation that becomes active at time t is:

$$U_t = \sum_{k=1}^{G} \Re^{k-1} \cdot \ln c_{g(k),t+k-1} + \Re^{G-1} \cdot \phi_{g(G)} \cdot \ln beq_{g(G),t+G-1} \tag{18}$$

where $\Re$ is an exogenous discount factor, c is consumption, and beq stands for bequests. U is maximised subject to:

$$\sum_{k=1}^{G} R_{t+k-1} \cdot \left(m_{g(k),t+k-1} - c_{g(k),t+k-1}\right) = R_{t+G-1} \cdot beq_{g(G),t+G-1} \tag{19}$$

where R_t is the market determined discount factor: $R_{t+k-1} = \prod_{s=t+1}^{t+k-1} \left(\frac{1}{1+r_s}\right)$ and $m_{g(k),t}$ is labour income net of social security contributions at rate τ_{sc}, pension benefits and inheritances:

$$\begin{aligned} m_{g(k),t} &= \sum_{i=1}^{I} \sum_{sex} (1-\tau_{sc}) \cdot A_{i,g(k),sex,t} \cdot w_{i,t} \cdot s_{g(k),sex,t} \cdot l_{i,g(k),sex,t} \\ &\quad + pens_{g(k),t} + inh_{g(k),t} \end{aligned}$$

where inh are the inheritances from uniformly distributed bequests to the youngest generation, $w_{i,t}$ the per unit of effective labour wage in profession i, $s_{g(k),sex,t}$ the proportion of males and females in the population by class age, $l_{i,g(k),sex,t}$ the proportion of professions by class age and sex ($l_{i,g(k),sex,t} = \frac{L_{i,g(k),sex,t}}{N_{g(k),t} \cdot s_{g(k),sex,t}}$ from equation (15)), and labour productivity $A_{i,g(k),sex,t}$ depends on age and on characteristics such as sex:

$$\ln A_{i,g(k),sex,t} = \varphi_{1,i} k + \varphi_{2,i} k^2 + \varphi_{3,i} sex \tag{20}$$

The economy produces one good in amount X using physical capital K and effective labour of different professions $\mathfrak{L}_i$ with a constant returns to scale Cobb-Douglas technology:

$$X_t = \prod_{i=1}^{I} \mathfrak{L}_{i,t}^{\alpha_i} \cdot K_t^{\beta}$$

A pension system is Pay-As-You-Go with fixed social security rate τ_{sc}, the replacement ratio γ being endogenously determined to ensure balanced social security budget at each t:

$$pens_{g(k),t} = \gamma_t \cdot \sum_{i=1}^{I} \sum_{sex} A_{i,g(k),sex,t} \cdot w_{i,t} \cdot s_{g(k),sex,t} \cdot l_{i,g(k),sex,t} \tag{21}$$

The capital stock accumulation depends on investments and on capital depreciation:

$$K_{t+1} = K_t \cdot (1 - \delta) + Inv_t \tag{22}$$

The price system ($w_{i,t}$, r_t) is determined so that markets balance at each time period:

$$X_t = \sum_k N_{g(k),t} \cdot c_{g(k),t} + Inv_t \tag{23}$$

$$\mathfrak{L}_{i,t} = \sum_k \sum_{sex} N_{g(k),t} \cdot A_{i,g(k),sex,t} \cdot s_{g(k),sex,t} \cdot l_{i,g(k),sex,t} \tag{24}$$

4 The dynamics of income distribution in an ageing population: an illustrative example

In this section, we wish to illustrate the usefulness of these results by evaluating potential effects of population ageing on the dynamics of income distribution and inequalities, using a simple OLG model when individuals have to make leisure/work decisions, and choose one of two possible professions (indicated by Prof-0 and Prof-1). Obviously, addressing such issues requires a consistent use of both the microsimulation set-up – to keep track of individuals – and the general equilibrium. For this, we shall use an artificial – though plausible – computer-generated micro data-set of 30.000 individuals, and link this to an applied OLG model calibrated on a fictitious – though by no means unrealistic – macro data-set that can be thought of as representative of some archetype OECD economy. Assuming the dynamic economy is initially in a steady state, we then submit it to a permanent (and unexpected) demographic slowdown.

4.1 The micro data-set

In this stationary population, we distinguish individuals by gender and age groups of ten years each, starting at age 15. Only those belonging to the first five age classes have discrete choices to make: to work or not to work, and in which profession. Those from the last three generations are exogenously retired from the labour force. There are 30.000 such decision-making individuals, each belonging to one specific cell of characteristics, in proportions conveyed by Table 1 in Appendix. Individual wages by profession are generated by using the Mincerian equations $\ln w_i^h = const_i + \alpha_{1i} \cdot age^h + \alpha_{2i} \cdot \left(age^h\right)^2 + \alpha_{3i} \cdot sex^h + \epsilon_i^h$. The parameters of the Mincerian equations are shown in Table 2.

Intra-cell individual heterogeneity in preferences is then generated using i.i.d. double exponential stochastic terms with inverse dispersion parameters $1/\mu$ and $1/\upsilon$ – which are the σ and τ substitution elasticities of (16) – reported in Table 3. Finally, the preference parameters are chosen so as to generate reasonable shares of workers/unemployed, as well as contrasted shares of professions: see Table 4. The average level and the standard deviation of wages are reported in Table 5.

4.2 The macro data-set and the ageing shock

In this illustrative simulation exercise, we assume the economy initially in a steady-state that is stationary. Because all individuals are assumed to exit at the same age of 95, the dependency ratio is rather high in this economy, at 60%. (We could have taken care of this by introducing mortality rates at each age but with little additional insight given illustrative-only ambition of the exercise.) The main parameters and data of the macro model are summarised in Table 6 in Appendix.

The ageing shock is implemented by a temporary drop of the parameter η_t with resulting population time-path displayed in Figure 7, and old-age dependency ratio (i.e., the ratio of retired to active population) as displayed in Figure 8. This is indeed a quite drastic ageing shock. The reason for choosing an admittedly excessive demographic change is that we want to ensure significant factor-price changes and hence, induce significant switches in individual discrete decisions: only then can we gain true confidence in our methodology. The resulting solution time-path of factor prices is displayed in Figure 9, and is as one expects.

4.3 Accuracy

Having computed the equilibrium path of wages, we now plug back these factor prices into the microsimulation model, and compute the new optimal discrete choices for each of the 30000 individuals, aggregate these per population cells and compare with those generated from the representative agent formulation in the OLG model. Why could these predictions differ, given that we use exact aggregation results? The reader will remember that, within each population cell, we assumed that the labour supply decision results from considering – both in the micro and in the macro approach – the Mincer-wage w_i rather than the true individual wage w_i^h which is w_i adjusted for within-cell idiosyncratic productivity differences. The ex-post microsimulation evaluation uses this individual information that has been lost in the aggregation process. Checking for these errors is therefore indeed meaningful.

Table 10 and Table 11 provide a sample of accuracy results, measured as % discrepancies between the two predicted labour supplies. Observe that the first time period discrepancies are all of the order of 1.E-10 which only reflects the quality of the calibration: indeed, the demographic shock only affects the economy at later periods. Looking at the time path of errors, we see that the largest is roughly equal to half a per cent, a very small number given the severity of the demographic shock: clearly, a discrepancy that is unlikely to affect the equilibrium wages and is therefore without GE implication.

4.4 Income inequalities induced by population ageing

We now report how the ongoing ageing of our economies may affect income inequalities, an issue that can now be rigorously addressed thanks to the microsimulations model. Among the various inequality indices, we choose two without apologies: our results are purely illustrative and do not require thorough dwelling.

We first report in Figure 12 the median, tenth percentile, and ninetieth percentile of the (net of social security contributions) total income distribution for the entire active population (that is, excluding the retired cohorts). The dynamics of the median and ninetieth percentile are easy to understand from the time path of wages (see Figure 9): the former individual is a young lower-skilled – i.e., working in the profession where wages are lowest – who benefits from increasing wages in Profession 0 and is largely unaffected by the drastic reduction in capital returns; the latter individual is an older – and hence with more accumulated assets – qualified worker (i.e., working in Profession 1) whose rising wage more than compensates depressed returns on capital during the first half of the time horizon, and whose recovering capital income offsets later the contracting labour earnings. Not surprisingly, the time profile of the tenth percentile of income is more erratic reflecting undiversified factor ownership in the lower tail of the income distribution: up to time period 8, the pivotal individual is a (possibly up to then unemployed) low-skilled worker who benefits from rising wages and is immune to fluctuations in capital returns, whereas for the next ten years, the pivotal individual, because unemployed, is strongly affected by low interest rates.

We end this section by reporting in Figure 13 and Figure 14 the contrasted time path of the Gini coefficients for age-groups 45-54 and 55-64 which of course are what we expect from what we know on the cohorts from Table 4 and from the factor-price movements (Figure 9).

5 Conclusion

Applied GE models have become indispensable tools of qualitative policy assessment. By essence, they rely on some form of representative agents' simplification of the economy so as to make explicit and manageable the consistency imposed on individual decisions by technological and resource constraints. As huge micro data-sets have increasingly been made available in recent years, the microsimulation approach has developed that apprehends the full heterogeneity of individual behavioural adjustments to policy reforms at the expense of global consistency. In these models, individual decision-making often is of discrete-choice type. In this paper, we suggested a bridge between these two approaches by making use of exact aggregation results due to Anderson, de Palma and Thisse (1992). We have argued that this provides an extremely useful interface between the two approaches: it makes possible to counter the major weakness of each of the two approaches making them consistently complementary. We have illustrated this in a dynamic setting, by linking a microsimulation model built from a computer-generated micro data-set to an OLG-GE representation of an economy submitted to demographic ageing.

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Appendix

Males	Number	*Females*	Number
g(1) : 15-24	3000	*g(1)* : 15-24	3000
g(2) : 25-34	3000	*g(2)* : 25-34	3000
g(3) : 35-44	3000	*g(3)* : 35-44	3000
g(4) : 45-54	3000	*g(4)* : 45-54	3000
g(5) : 55-64	3000	*g(5)* : 55-64	3000
total	15000	total	15000

Table 1: Number of individuals by age and sex

	Prof-0	Prof-1
constant	5	6
age	0.3	0.45
age x age	-0.005	-0.003
sex	-0.4	-0.35

Table 2: Parameters of the Mincerian equations

	Leisure / Work	Prof-0 / Prof-1
Males		
g(1) : 15-24	1.752	2.336
g(2) : 25-34	1.753	1.177
g(3) : 35-44	1.739	2.527
g(4) : 45-54	1.749	1.449
g(5) : 55-64	1.727	1.824
Females		
g(1) : 15-24	1.759	2.397
g(2) : 25-34	1.772	2.226
g(3) : 35-44	1.789	1.827
g(4) : 45-54	1.742	1.961
g(5) : 55-64	1.755	1.417

Table 3: Substitution elasticities

	Unempl / Total	Workers / Total	Prof-0 / Workers	Prof-1 / Workers
Males				
g(1) : 15-24	17.47%	82.53%	61.79%	38.21%
g(2) : 25-34	15.33%	84.67%	74.61%	25.39%
g(3) : 35-44	14.60%	85.40%	51.80%	48.21%
g(4) : 45-54	12.67%	87.33%	58.59%	41.41%
g(5) : 55-64	25.47%	74.53%	51.03%	48.97%
Females				
g(1) : 15-24	18.47%	81.53%	62.18%	37.82%
g(2) : 25-34	31.43%	68.57%	57.07%	42.93%
g(3) : 35-44	15.87%	84.13%	65.17%	34.83%
g(4) : 45-54	15.43%	84.57%	62.83%	37.17%
g(5) : 55-64	22.50%	77.50%	57.59%	42.41%

Table 4: Unemployment rates, activity rates, and activity rates by profession

	Mean	Stand dev
Males	779.505	907.928
Females	498.879	557.351
g(1) : 15-24	213.643	192.866
g(2) : 25-34	334.056	266.025
g(3) : 35-44	609.024	497.760
g(4) : 45-54	850.263	741.632
g(5) : 55-64	1188.975	1186.759

Table 5: Mean and standard deviation of wages

Consumption / GDP	80%
Investments / GDP	20%
Gross capital remuneration / GDP	33.3%
Remuneration of *Prof-0* / GDP	24.9%
Remuneration of *Prof-1* / GDP	41.8%
Social security contributions	20%
Gross interest rate	8.3%
Depreciation rate	5.0%
Intertemporal substitution elasticity	1

Table 6: Main parameters used in the OLG model

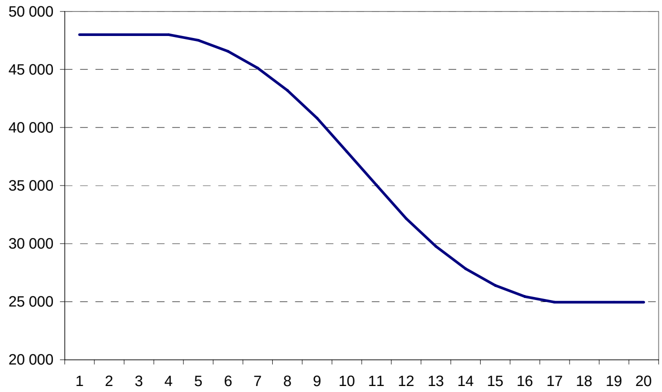

Figure 7: Population

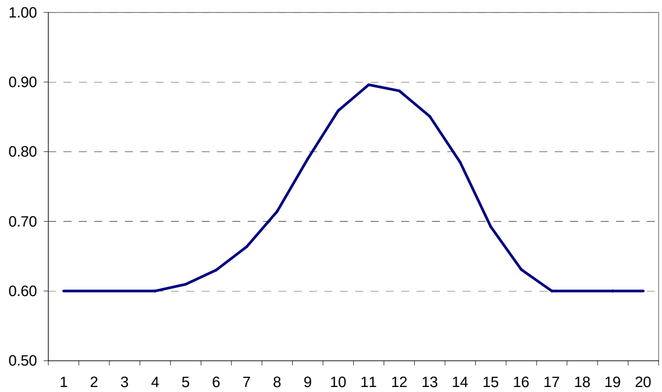

Figure 8: Old-age dependency ratio

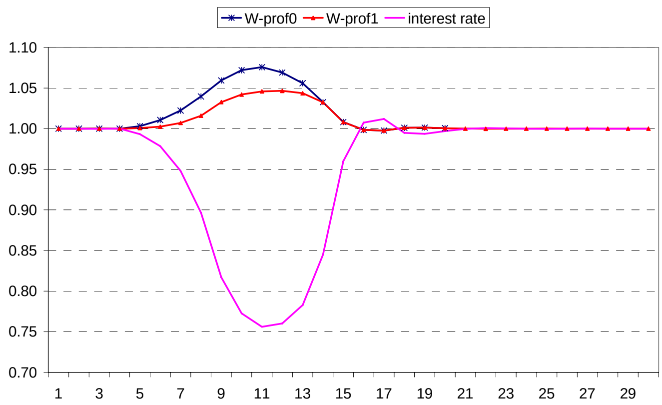

Figure 9: Factor prices

	g(1)	g(2)	g(3)	g(4)	g(5)
1	-1.17684E-10	5.55E-11	2.24265E-10	2.5091E-10	4.04121E-10
2	-2.16404E-06	-3.09893E-07	-1.64776E-06	-3.75061E-07	-8.0231E-07
3	-4.9683E-06	-7.1157E-07	-3.78319E-06	-8.6116E-07	-1.8421E-06
4	1.97866E-06	2.83484E-07	1.5069E-06	3.43072E-07	7.33646E-07
5	-0.056429459	-0.007454646	-0.039563773	-0.009020953	-0.019281744
6	-0.203157036	-0.026820182	-0.144885989	-0.033181634	-0.0707877
7	-0.37510605	-0.049358284	-0.256597097	-0.060636206	-0.129106824
8	-0.532531217	-0.076389833	-0.417204573	-0.093814143	-0.19607371
9	-0.584325694	-0.084696476	-0.441801666	-0.106988847	-0.186627112
10	-0.628763647	-0.097380121	-0.498288113	-0.124620973	-0.188435987
11	-0.621275487	-0.092177424	-0.530580951	-0.117289375	-0.180726575
12	-0.473838398	-0.07001143	-0.39782689	-0.089091283	-0.152625865
13	-0.263820653	-0.038758399	-0.221720741	-0.046068173	-0.064186342
14	-0.002635106	-0.00038454	-0.002217275	-0.000457406	-0.00051224
15	-0.000675913	-9.86695E-05	-0.000568882	-0.000117413	-0.00013115
16	0.00012707	1.85523E-05	0.00010696	2.208E-05	2.46377E-05
17	0.000202767	2.96044E-05	0.000170678	3.52341E-05	3.93118E-05
18	-8.86393E-05	-1.29408E-05	-7.46087E-05	-1.54008E-05	-1.71897E-05
19	-0.000107553	-1.57021E-05	-9.05284E-05	-1.86869E-05	-2.0858E-05
20	-4.98291E-05	-7.27476E-06	-4.19418E-05	-8.65777E-06	-9.66293E-06

Table 10: % discrepancies: labour supply by males

	g(1)	g(2)	g(3)	g(4)	g(5)
1	-8.31557E-10	8.88E-11	3.06422E-10	-3.51941E-10	-2.55351E-10
2	-2.1646E-06	-3.09953E-07	-1.64796E-06	-3.75182E-07	-8.02445E-07
3	-4.96885E-06	-7.11654E-07	-3.78341E-06	-8.61299E-07	-1.84221E-06
4	1.97811E-06	2.83407E-07	1.50673E-06	3.42919E-07	7.33544E-07
5	-0.05642946	-0.007454647	-0.039563773	-0.009020954	-0.019281744
6	-0.203157036	-0.026820182	-0.144885989	-0.033181634	-0.0707877
7	-0.375106051	-0.049358284	-0.256597097	-0.060636206	-0.129106824
8	-0.532531218	-0.076389833	-0.417204573	-0.093814143	-0.196073711
9	-0.584325694	-0.084696476	-0.441801667	-0.106988848	-0.186627112
10	-0.628763648	-0.097380121	-0.498288113	-0.124620973	-0.188435987
11	-0.621275487	-0.092177424	-0.530580951	-0.117289375	-0.180726576
12	-0.473838399	-0.07001143	-0.397826891	-0.089091283	-0.152625865
13	-0.263820654	-0.038758399	-0.221720741	-0.046068173	-0.064186342
14	-0.002635107	-0.00038454	-0.002217275	-0.000457406	-0.00051224
15	-0.000675914	-9.86696E-05	-0.000568882	-0.000117413	-0.00013115
16	0.00012707	1.85522E-05	0.00010696	2.20799E-05	2.46376E-05
17	0.000202766	2.96043E-05	0.000170678	3.5234E-05	3.93117E-05
18	-8.86398E-05	-1.29409E-05	-7.46089E-05	-1.54009E-05	-1.71898E-05
19	-0.000107554	-1.57022E-05	-9.05287E-05	-1.8687E-05	-2.08581E-05
20	-4.98296E-05	-7.27482E-06	-4.1942E-05	-8.65787E-06	-9.66304E-06

Table 11: % discrepancies: labour supply in profession 1 by males

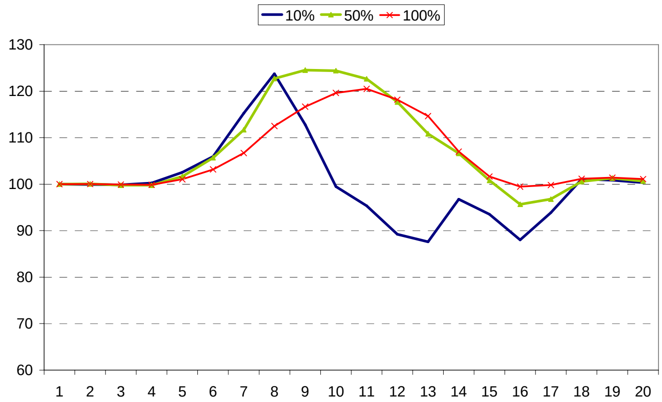

Figure 12: Evolution of the 10^{th}, 50^{th}, and 90^{th} percentiles

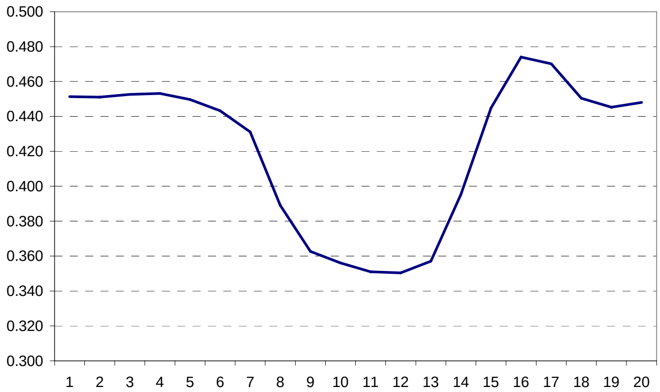

Figure 13: Gini index for the age group 45-54

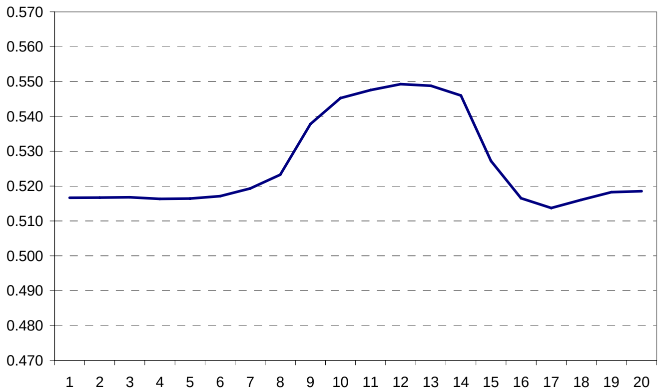

Figure 14: Gini index for the age group 55-64

On income inequality effects of population ageing: an evaluation using an integrating micro-macro simulation approach

Abstract

This paper provides a step further towards linking dynamic general equilibrium models and microsimulation models by using the exact aggregation theory due to Anderson, de Palma and Thisse (1992) that allows aggregating individual optimal decisions into explicit labour supply functions that could be introduced in an OLG model. We apply this methodology on the Canadian economy in order to analyse the potential impacts of population ageing on income inequalities.

This paper is joint with Jean Mercenier, THEMA - Université de Cergy-Pontoise, 33 Bd du Port, 95011 Cergy-Pontoise Cedex.

1 Introduction

Applied GE models have become indispensable tools of qualitative policy assessment. By essence, they rely on some form of representative agents' simplification of the economy so as to make explicit and manageable the consistency imposed on individual decisions by technological and resource constraints. As huge micro data sets have increasingly been made available in recent years, the micro simulation approach has developed that apprehends the full heterogeneity of individual behavioural adjustments to policy reforms at the expense of global consistency. In these models, individual decision-making often is of discrete-choice type. In a companion paper (Magnani and Mercenier, 2006), we have shown how exact aggregation results due to Anderson, de Palma and Thisse (1992) provide a natural bridge between the two policy tools. For this purpose, we used a computer-generated micro data set to model individual nested discrete-choices from heterogeneous agents having to make leisure/work decisions, and choose between two professions. Their individual optimal decisions were then aggregated into explicit labour supply functions, and used in a simple OLG model to evaluate the potential effects of population ageing on the dynamics of income distribution in a plausible though fictitious economy.

In this paper, we go one step further, and apply the methodology on real world data from the Canadian economy. The step is not a minor one, though: the nested multinomial logit systems will now have to be statistically estimated from the data set whereas in our previous paper the individual data were artificially generated conditional on arbitrarily chosen parameters of the discrete-choice preferences. Realism also requires that the discrete-choice set be expanded from two to ten professions. Furthermore, we will endogenise the young generation's educational choices assuming heterogeneous individual preferences over a discrete set of education alternatives. Finally, the OLG model will be calibrated on Canadian economy and demographics and used to apprehend some ageing effects on the Canadian economy.

Our paper provides an interesting complement to a recent paper by Mercenier, Mérette and Fougère (2004), hereafter MMF2004. In that paper they investigate the sectoral and skill/occupational impact of population ageing by taking into account, in an applied OLG model, both the supply-side and the demand-side channel by which the demographic shock will impact on the Canadian labour market. They however choose to neglect market-induced changes in the characteristics of the labour supply: labour supply is held exogenous and its composition in terms of occupations and qualifications is assumed to remain identical across households. Our work precisely focuses on modelling the behaviour of skill and occupational characteristics of the Canadian labour supply.[1]

The paper is organised as follows. In Section 2, we provide a refresher on the theoretical framework concerning the aggregation result emerging from assuming multinomial logit heterogeneity in preferences. In Section 3 we provide a description of the Canadian data set used in our analysis, while in Section 4 we present the results of our estimations concerning, on the one hand, the nested multinomial logit estimation "work-leisure" and choice of the profession, and on the other hand, the multinomial logit estimation of the choice of education. Section 5 provides a short description of the OLG model. We then simulate in Section 6 the ageing shock on the Canadian economy, and plug into individual decision problems the computed equilibrium prices to evaluate the effect of population ageing on the dynamic path of income inequality indicators. We conclude in the last section.

2 The theoretical framework

2.1 The leisure/work decision and the choice of a profession

2.1.1 The discrete choice formulation

We model the "leisure/work in which profession" decision as resulting from maximising nested two-level multinomial logit preferences.[2]

Let h be an individual belonging to a population-cell grouping those who share the same characteristics: age, sex, and education level; and let there be I possible professions indexed i, j. We write the utility h gets from each profession as a log-linear function:

$$\widetilde{v}_i^h = \ln \theta_i + \beta \ln w_i + \epsilon_i^h \qquad i = 1, ..., I$$

[1] Our clustering of professions and education levels is basically similar to theirs - as is our definition of generations - which will make comparisons meaningful.

[2] We here only provide a sketch of the discrete-choice decision problem. The reader is referred to our companion paper (Magnani and Mercenier, 2006) for a detailed exposition.

where the first term captures the (common to all professions) disutility of working as well as the welfare costs/benefits of various characteristics specific to profession i, and w_i is the adjusted for characteristic-specific efficiency wage expressed in terms of the consumption good. In the rest of the paper, we shall refer to this wage as the Mincer wage of that cell. These two terms are assumed common to all h within the considered population cell[3]. Intra-cell individual heterogeneity in preferences is captured by the i.i.d. double exponential stochastic term ϵ_i^h.[4] The probability that individuals belonging to that cell will choose profession i is:

$$P_i = \frac{\exp(\ln\theta_i + \beta\ln w_i)}{\sum_j \exp(\ln\theta_j + \beta\ln w_j)} = \frac{\theta_i \cdot w_i^\beta}{\sum_j \theta_j \cdot w_j^\beta}$$

This probability is of course conditional on the individual deciding to work rather than to leisure, which we now describe.

We write the utility h enjoys from not working as:

$$\widetilde{V}_0^h = \ln\Theta_0 + \varepsilon_0^h$$

where ε_0^h is a random term which captures individual heterogeneity in the valuation of leisure (the disutility of working). The alternative is for the individual to work, taking into account that if he does so, he will be able to choose the best profession. The valuation of the alternative *work* that is consistent with the second stage decision problem is:

$$\widetilde{V}_1^h = \beta_1 V_1 + \varepsilon_1^h$$

[3] We therefore assume here that, upon making their optimal decisions, individuals ignore possible within-cell idiosyncratic productivity differences, that will ex-post be responsible for the observed distribution of wages in the data. The additional information contained in the within-cell distribution of individual wages w_i^h will be used in the calibration of the general equilibrium model, in the econometric estimation of the parameters of the discrete-choice preferences, and in the ex-post microsimulations.

[4] In our previous paper (Magnani and Mercenier, 2006), we used the following notation: $\widetilde{u}_i^h = const_i + \ln w_i + \varepsilon_i^h$ where ε_i^h are i.i.d. double exponential with mean equal to $\mu \cdot \gamma$, where γ is the Euler's constant (≈ 0.577), and dispersion parameter μ, so that the variance is given by $\frac{\pi^2\mu^2}{6}$. Consider the new notation $\widetilde{v}_i^h = \ln\theta_i + \beta\ln w_i + \epsilon_i^h$ and suppose that the mean of ϵ_i^h is γ and the variance $\frac{\pi^2}{6}$. It is easy to see that the two expressions coincide if $\beta = \frac{1}{\mu}$. In fact, by dividing the new expression by β we find $\frac{\widetilde{v}_i^h}{\beta} = \frac{\ln\theta_i}{\beta} + \ln w_i + \frac{1}{\beta}\epsilon_i^h$, where $\frac{1}{\beta}\epsilon_i^h$ are double exponential distributed with mean $\frac{\gamma}{\beta}$ and variance $\frac{\pi^2}{6\beta^2}$.

where V_1 is the expected maximum utility of a subset of alternatives:

$$\begin{aligned} V_1 &= \frac{1}{\beta} \cdot \ln \sum_i \exp\left(\ln \theta_i + \beta \cdot \ln w_i\right) \\ &= \frac{1}{\beta} \cdot \ln \sum_i \theta_i \cdot w_i^{\beta} \end{aligned} \tag{1}$$

Assuming $\varepsilon_0^h, \varepsilon_1^h$ are double exponential i.i.d. random terms, the probability of choosing to work (option 1) can be shown to be:

$$P_1 = \frac{\left[\sum_i \theta_i \cdot w_i^{\beta}\right]^{\beta_1/\beta}}{\Theta_0 + \left[\sum_i \theta_i \cdot w_i^{\beta}\right]^{\beta_1/\beta}}$$

2.1.2 Aggregation of individual choices and the representative agent formulation

There are N individuals in a population cell with identical characteristics, N large enough. The within-cell aggregate labour supply in each profession resulting from individual discrete choices is then easily established as:

$$\begin{aligned} L_i &= P_i \cdot P_1 \cdot N \\ &= \frac{\theta_i \cdot w_i^{\beta}}{\sum_j \theta_j \cdot w_j^{\beta}} \cdot \frac{\left[\sum_j \theta_j \cdot w_j^{\beta}\right]^{\beta_1/\beta}}{\Theta_0 + \left[\sum_j \theta_j \cdot w_j^{\beta}\right]^{\beta_1/\beta}} \cdot N \quad i = 1, \cdots, I \end{aligned} \tag{2}$$

It can easily be shown (see Magnani and Mercenier, 2006) that this expression can be derived from an optimising representative agent who sequentially determines first its preferred time split between leisuring and working, and next its sharing of working time between professions, using two constant elasticities of transformation function with parameters related to those of the discrete-choice formulation as follows:

$$\begin{cases} \sigma = \beta \\ \alpha_i^{-\sigma} = \theta_i \end{cases} \quad \text{and} \quad \begin{cases} \tau = \beta_1 \\ \alpha_{\mathcal{L}}^{-\tau} = \Theta_0 \end{cases} \tag{3}$$

where σ and τ are the constant elasticities of transformation and the αs are the share parameters of the representative agent's preferences.

These parameters can therefore be estimated using the micro data and discrete choice econometric techniques, and the resulting aggregate labour-supply systems plugged into the general equilibrium model.

2.2 Choosing education

Individuals from the younger generation face an additional specific choice problem: they have to choose which education level to achieve. There are two small difficulties here that we have to deal with before being able to apply the same discrete-choice methodology as in the previous section.

Firstly, education degrees are not mutually exclusive alternatives as is typically assumed in the discrete-choice model. This in particular implies that we cannot infer from the data if an individual in the process of acquiring some education degree targets this as its ultimate ambition, or if his current education efforts constitute only an intermediate step towards higher achievements. We solve this problem by only considering from the data set, and for the purpose of econometric estimation, those individuals that are aged 24, that is, the oldest of the younger generation. Indeed, it seems reasonable to assume that after that age, only very few individuals will still be moving up the education ladder. We can thus use this population subgroup to estimate the choice probabilities for education using the standard discrete-choice formulation between mutually exclusive alternatives.

The second difficulty arises from the intertemporal nature of education choices: individuals make their education choices taking into account the extra earnings they expect to gain from additional qualifications. This information is of course not provided by the data. In other words, prior to the statistical estimation of the discrete-choice model for education, we have to generate some expected relative education returns that will then be used in the estimation process.

Let us index possible education levels by indices edu, edu'. We compute for each education degree a discounted sum of future earning streams, making use of the choice probabilities between professions estimated from the previous subsection.

Formally, we define H_{edu} as the asset-value of the education level edu:[5]

$$H_{edu} = \sum_{age=1}^{5} (1+r)^{-(age-1)} \cdot \psi_{age,edu} \cdot \mathrm{E}[w_{age,edu}]$$

with r is the interest rate, $\psi_{age,edu}$ the amount of time h can work in each period (that is, a decade for all working generations except for those young who choose more or less time consuming education plans) and $\mathrm{E}[w_{age,edu}]$ the mathematical expectation of anticipated future wages:

$$E[w_{age,edu}] = \sum_{i=1}^{I} P_{i,age,edu} \cdot w_{i,age,edu}$$

As a first step, we assume here[6] that the valuation made by an individual h aged 15-24 of an education degree is then:

$$V_{edu}^{h} = \ln \Theta_{edu} + \beta_2 \ln H_{edu} + \varepsilon_{edu}^{h}$$

where ε_{edu}^{h} are i.i.d. double exponential distributed.

The probability of choosing a specific education degree then writes as follows:

$$P_{edu} = \frac{\Theta_{edu} \cdot H_{edu}^{\beta_2}}{\sum_{edu'} \Theta_{edu'} \cdot H_{edu'}^{\beta_2}}$$

With N individuals in the population cell, N large enough, we can then establish the proportion that will choose to achieve each education degree:

$$\frac{N_{edu}}{N} = \frac{\Theta_{edu} \cdot H_{edu}^{\beta_2}}{\sum_{edu'} \Theta_{edu'} \cdot H_{edu'}^{\beta_2}} \qquad edu = edu1,\ edu2, ... \tag{4}$$

[5] In order to ease notation, we neglect in what follows the gender index.

[6] There are many reasons why this definition of the asset-value of an education degree is not satisfactory and will require additional work. First it seems that individuals would take account of the probability they face to exit the labour market in the future when computing the expected labour earning, something neglected here. Also, the current formula does not take into account the impact of education on pension benefits to be earned. The third reason is more subtle and complex to be dealt with: H_{edu} is an ingredient of the individual discrete-choice problem, and its computed values are to be used in the multinomial logit statistical estimation of preference parameters that will condition the future time path of the OLG economy. This future time path of aggregate variables will in turn feed back into the discrete-choice probabilities through the individual valuation of assets. We therefore face here a complex fixed point that involves iterating between the dynamic macro model and the discrete-choice estimation procedure at the micro level. Future work will have to establish the convergence properties of such an iterative procedure to solve this fixed point problem.

3 The micro data set and Mincer wages

3.1 The data set

Our data set is the FMGD (*Fichier de microdonnées à grande diffusion*) of the 2001 population survey conducted by Statistics Canada, designed to provide a detailed description of the demographic, social and economic characteristics of the Canadian population. The data set originally includes a little more than 600000 individuals, of which we randomly draw a 10% sample: there are 62077 individuals of which 47133 are workers and 14944 are unemployed. We group individuals according to 5 age groups of ten years, 10 profession types and five education levels. We provide a comprehensive list of the professions and education levels as well as their definitions in Tables 1 and 2.[7] Tables 3 to 6 report some statistics about labour participation. In particular, Table 3 reports details about the occupation rates for all individuals, for females, for males and for different age groups, while Table 4 indicates the composition of each profession by sex and by age group. Table 5 reports the occupation rates for different education levels, and Table 6 indicates the composition of each profession by education level. More detailed tables are provided in Appendix B.

3.2 Mincer wages

The working population is thus divided into cells according to characteristics: age group, sex, education level and profession. We shall need the Mincer (or within-cell average) wage w_i for each cell. This we obtain by regressing, separately for each profession, the log of individual wages on exogenous characteristics. Formally:

$$\ln w_i^h = \alpha_{0i} + \alpha_{1i} \cdot age^h + \alpha_{2i} \cdot \left(age^h\right)^2 + \alpha_{3i} \cdot sex^h + \alpha_{4i} \cdot edu^h + \kappa_i^h \qquad i = 1, ..., 10 \quad (5)$$

where age=1 if h belongs to the 15-24 age group, age=2 if he belongs to the 25-34 etc.; sex=1 for women and sex=2 for men; edu=1 for education level 1, edu=2 for education level 2 etc.; κ_i^h is a random term normally distributed with zero mean and standard error σ_i.

[7] MMF2004 refer to what we call "professions" and "education levels" as "occupations" and "qualification levels".

The results of these estimations are reported in Table 7. We note, in particular, the inverse U-shaped age profile of wages, as one expects, and as illustrated in Figures 8a and 8b for individuals with different education levels. We contrast in Figure 9 our results, averaged over all individuals, with the (identical for all) earnings profile used by MMF2004: it seems that they drastically underestimate the importance of experience on individual labour productivity. We also note, from the coefficient α_{3i} , the % wage gap due to gender differences: as is no surprise, men earn between 28 and 66 % more than women at equal age and education, depending on the profession. The per-education average wage premium varies between professions between 3% and 26%, a somewhat modest number.

The Mincer wage for each population cell is then computed as:[8]

$$\ln w_i = \widehat{\alpha}_{0i} + \widehat{\alpha}_{1i} \cdot age + \widehat{\alpha}_{2i} \cdot (age)^2 + \widehat{\alpha}_{3i} \cdot sex + \widehat{\alpha}_{4i} \cdot edu \qquad i = 1, ..., 10 \qquad (6)$$

4 The model estimation

4.1 The nested two-level multinomial logit estimation

Given the size of our data set, the nested choice "work-leisure" and the choice of profession can equivalently be estimated (by Maximum Likelihood) either jointly or by a two-step single equation procedure. We adopt the second approach that yields consistent estimates and is computationally much simpler (see Amemiya, 1985, p303).

4.1.1 The choice of a profession

Generating individual perceived potential wage Obviously, the micro data provides us with information on wages earned by individuals in their chosen profession, and not on the wage they would have earned had they made a different choice. In other words, to estimate individual preferences, it is necessary to generate "perceived potential wage" for the professions that are not chosen. We do this as follows.

Consider h has chosen profession i. We can compute the percentage gap (noted θ^h) between his observed wage w_i^h and w_i, the Mincer wage corresponding to the same char-

[8] Given that the Mincer wage depends on age, sex, and education level, it should rigorously be written as $w_{i,age,sex,edu}$. We simplify the notation by writing w_i, as opposed to w_i^h; we believe that no ambiguity will result.

acteristics:

$$\theta^h = \ln w_i^h - \ln w_i \tag{7}$$

A positive θ^h expresses an above within average productivity for individual h in the chosen profession. It is then reasonable to assume that *(a)* on average, everything else equal, an individual has chosen the profession for which his θ^h is highest, and *(b)* that on average, the sign of an individual's θ^h is invariant with the profession. We then attribute to the non-chosen professions $j \neq i$ a "perceived potential wage" generated from the following equation:

$$\ln w_j^h = \ln w_j + \gamma_j \cdot \theta^h \tag{8}$$

$$\gamma_j = N(\overline{\gamma}_j, \widehat{\sigma}_j) \tag{9}$$

with $\overline{\gamma}_j = \begin{cases} 0.5 \text{ if } \theta^h > 0 \\ 1.5 \text{ if } \theta^h < 0 \end{cases}$ and $\widehat{\sigma}_j$ is the standard error of the Mincer regression for profession j.

Estimating the preference parameters for profession We are now in a position to proceed, for each sex and education level, to the statistical estimation of the preference parameters governing individual professional choices:

$$\widetilde{v}_i^h = \ln \theta_i + \beta \cdot \ln w_i^h + \epsilon_i^h \tag{10}$$

where $\widetilde{v}_i^h$ is the utility enjoyed by individual h from choosing profession i, and the ϵ_i^h are i.i.d. double exponentially distributed random terms.

The results of this estimation (we only report the $\widehat{\beta}$s) are reported in Table 10. These coefficients are equal to the characteristic-specific transformation elasticities between professions of the aggregate agent. Observe that all estimates are statistically quite accurate since they are significant at 1% level. All fall in the (0.5, 1.3) interval though many are lowers than unity. As one expects, young adults are more mobile between professions than older ones. When young, women tend to change profession more easily than men, but become relatively less reactive than males as they age. Interestingly, at all ages, more educated women tend to change occupations more easily than those who are less educated, though this does not seem to be true for men. Even though these transformation elasticities may be somewhat lower than one would expect, they however clearly prove that interprofessional mobility matters and should be acknowledged for in an ageing context.

4.1.2 Choosing to work or to leisure

As we already mentioned, the utility that h enjoys from not working and working are respectively:

$$\begin{cases} \widetilde{V}_0^h = \ln \Theta_0 + \varepsilon_0^h \\ \widetilde{V}_1^h = \beta_1 \cdot \widehat{V}_1^h + \varepsilon_1^h \end{cases}$$

For those individuals in the data set who do work, we know that the opportunity cost of leisure is given by (1) where $\widehat{V}_1^h$ is computed as follows:

$$\widehat{V}_1^h = \frac{1}{\widehat{\beta}} \cdot \ln \sum_i \exp\left(\ln \widehat{\theta}_i + \widehat{\beta} \cdot \ln w_i^h\right)$$

For those who choose to leisure, however, we face a statistical problem: we have no information on the wages they would earn had they made the opposite choice. We could assume that a good proxy for this is provided by the Mincer wage, but as we know from Heckman's 1977 seminal intuition, this would result in a selection bias that therefore requires correction.

Consider the parameter β_1. The correlation coefficient between the residuals in the regression of the choice of profession (ϵ_i^h) and those in the regression of the work-leisure choice (ε_0^h and ε_1^h) is given by $1-\left(\frac{1}{\beta_1}\right)^2$ (see Amemya, 1985, p300). In particular, if $\beta_1 = 1$, the residuals are independent, so that the two nested discrete choices can be estimated by a single multinomial logit model. This would imply that unemployed people behave in the same way as those who work and therefore that their wages are generated from the same distribution in which case there is no selection bias. This is seldom the case in applications: the estimated parameter β_1 is generally lower than one. The selection bias has therefore to be taken care of. The strategy is then to determine the distribution of wages across professions for unemployed people imposing that this selection bias vanishes. To do this, we use the previously estimated Mincer wages for each profession adjusted for an error term Normally distributed, whose standard error is estimated from the Mincer regression residuals (the $\widehat{\sigma}_i$) and whose (expected to be negative) mean value $\widehat{m}_i$ is computed so that the estimated β_1 parameter be equal to one. In other words, this procedure applies to the unemployed a shifted distribution of Mincer wages estimated from those who actually work.

The wage distribution associated to each unemployed is then:

$$\ln w_i^h = \ln w_i + \widehat{\varepsilon}_i^h \qquad i = 1, \cdots, 10 \tag{11}$$

with $\widehat{\varepsilon}_i^h = N(\widehat{m}_i, \widehat{\sigma}_i)$. Table 11 reports on the value of the parameters $\widehat{m}_i$ which we refer to as the estimated wage penalty of the unemployed. Note that, with the exception of the lower education level, this penalty is always larger for women.

4.2 The choice of education

Table 12 reports the estimated transformation elasticity between qualifications degrees, obtained from individual discrete-choice logit on a pooled population of young men and women. Though statistically quite accurate, this coefficient should not be taken too seriously into account at this stage, since it is built conditional on a definition of the asset-value of education that is questionable, and calls elaborations as mentioned in note 5.

5 The OLG set-up

We now have, for each population cell, a different labour-supply system generated from aggregation of individual discrete choices, that is, there are as many representative labour-supplying agents as there are socioeconomic characteristics of interest in the micro database. We adopt identical and homothetic intertemporal preferences for these representative labour-supplying agents so that we can aggregate further these agents into a single (per-generation) representative consumer that optimally allocates its human wealth to lifetime consumption.

We distinguish between G generations that coexist at each time period t. At the end of each period, (a) the oldest group $g(G)$ disappears, (b) a new generation $g(1)$ enters the active population, and (c) a fraction of those belonging to the other age groups disappears according to exogenous mortality rates.

Each agent maximises its intertemporal utility subject to its wealth constraint. Doing so, he chooses: (a) the intertemporal profile of consumption (and therefore of asset accumulation); (b) how much to work, and in which profession (for those generations that are active, retirement is exogenously fixed at some late age). Formally, lifetime utility for the generation that becomes active at time t is:

$$U_t = \sum_{k=1}^{G} \Re^{k-1} \cdot \ln c_{g(k),t+k-1} \cdot \Omega_{g(k),t+k-1} \tag{12}$$

where $\Re$ is an exogenous discount factor, c is consumption, and $\Omega_{g(k),t}$ is the probability that an individual is still alive in t.[9]

U is maximised subject to the intertemporal budget constraint:

$$\sum_{k=1}^{G} R_{t+k-1} \cdot \left(m_{g(k),t+k-1} - c_{g(k),t+k-1}\right) = 0 \tag{13}$$

where R_t is the market determined discount factor: $R_{t+k-1} = \prod_{s=t+1}^{t+k-1} \left(\frac{1}{1+r_s}\right)$, and $m_{g(k),t}$ is labour income net of social security contributions at rate τ_{sc}, pension benefits and inheritances:

$$\begin{aligned} m_{g(k),t} \;=\; & \sum_{i} \sum_{sex} \sum_{edu} (1-\tau_{sc}) \cdot A_{i,g(k),sex,edu,t} \cdot w_{i,t} \cdot s_{g(k),sex,t} \cdot e_{g(k),sex,edu,t} \cdot l_{i,g(k),sex,edu,t} \\ & + pens_{g(k),t} + inh_{g(k),t} \end{aligned}$$

where inh are the inheritances from uniformly distributed bequests to the youngest generation, $w_{i,t}$ the per unit of effective labour wage in profession i, $s_{g(k),sex,t}$ the proportion of males and females in the population by class age, $e_{g(k),sex,edu,t}$ the proportion of education degrees by class age and sex ($e_{g(k),sex,edu,t} = \frac{N_{g(k),sex,edu,t}}{N_{g(k),t} \cdot s_{g(k),sex,t}}$ from equation (4)), $l_{i,g(k),sex,edu,t}$ the proportion of professions by class age, sex and education level ($l_{i,g(k),sex,edu,t} = \frac{L_{i,g(k),sex,edu,t}}{N_{g(k),t} \cdot s_{g(k),sex,t} \cdot e_{g(k),sex,edu,t}}$ from equation (2)), and labour productivity $A_{i,g(k),sex,edu,t}$ depends on age and on characteristics such as sex and the education level:

$$\ln A_{i,g(k),sex,edu,t} = \widehat{\alpha}_{1i} \cdot k + \widehat{\alpha}_{2i} \cdot k^2 + \widehat{\alpha}_{3i} \cdot sex + \widehat{\alpha}_{4i} \cdot edu \tag{14}$$

[9] Note that we do not introduce voluntary bequests in this problem but assume that all bequests are due to unexpected death. These involuntary bequests are then redistributed to the younger generation through inheritances in the budget constraint.

The economy produces one good in amount X using physical capital K and effective labour of different professions $\mathfrak{L}_i$ with a constant returns to scale Cobb-Douglas technology:

$$X_t = \prod_{i=1}^{I} \mathfrak{L}_{i,t}^{\alpha_i} \cdot K_t^{1-\sum_i \alpha_i}$$

A pension system is Pay-As-You-Go with fixed social security rate τ_{sc}, the replacement ratio γ being endogenously determined to ensure balanced social security budget at each t:

$$pens_{g(k),t} = \gamma_t \cdot \sum_i \sum_{sex} \sum_{edu} A_{i,g(k),sex,edu,t} \cdot w_{i,t} \cdot s_{g(k),sex,t} \cdot e_{g(k),sex,edu,t} \cdot l_{i,g(k),sex,edu,t} \quad (15)$$

The capital stock accumulation depends on investments and on capital depreciation:

$$K_{t+1} = K_t \cdot (1-\delta) + Inv_t \quad (16)$$

The price system $(w_{i,t},\, r_t)$ is determined so that markets balance at each time period:

$$X_t = \sum_k N_{g(k),t} \cdot c_{g(k),t} + Inv_t \quad (17)$$

$$\mathfrak{L}_{i,t} = \sum_k \sum_{sex} \sum_{edu} N_{g(k),t} \cdot A_{i,g(k),sex,edu,t} \cdot s_{g(k),sex,t} \cdot e_{g(k),sex,edu,t} \cdot l_{i,g(k),sex,edu,t} \quad (18)$$

5.1 The demographics in the model

In the model, the demographic evolution is described by the following equations:

$$\begin{aligned} N_{g(1),t+1} &= \eta_{,t} \cdot N_{g(1),t} \quad (19) \\ N_{g(k+1),t+1} &= N_{g(k),t} \cdot \gamma_{g(k+1),t+1} \end{aligned}$$

The first equation indicates that the number of people who belong to the first age group (20-24) in $t+1$ depends on the exogenous gross reproduction rate η_t. The second equation indicates that the number of people belonging to the age group $g\,(k+1)$ in $t+1$ depends on the number of people belonging to the age group $g\,(k)$ in t and on the exogenous survival probability $\gamma_{g(k+1),t+1}$.[10] Those demographic parameters are calibrated so as to reproduce the Canadian demographic history and projected future up to 2051. After that date, we assume that the old-age dependency ratio will gradually stabilise at 0.30 level.

[10] Of course Ω in equation (12) and γ in equation (19) are linked by the following relationship: $\Omega_{g(k),t} = \prod_{w=1}^{k} \gamma_{g(w),t-k+w}$.

The quality of the demographic calibration is summarised in Figures 13 and 14 in which we report the evolution of the old-age dependency ratio and the total population.

6 Simulation results

6.1 Macroeconomic effects of population ageing

Table 15 reports on the impact of population ageing on the macro indicators. The results are presented as time-series of indices with 2001 as base year.

Though per worker GDP increases during the first four decades, as a larger share of the working force has high productivity due to experience, it is already on the decline by 2051. The per capita GDP, however, displays a gloomier picture due to the increase in the inactive component of the population: the projected decline is steady during the first fifty years of the century: on average, Canadians have lost 10% of their living standards by year 2051. Though this last number is close to the one predicted by MMF2004, the time profile of per capita GDP is quite different: in their projections, the first two decades are characterised by modest but positive growth followed by a drastic 5% drop every 10 years between 2030 and 2050.

Most interesting are presumably the results on wages and on the interest rate: as one expects, the demographic choc boosts up the price of labour. In terms of the *numeraire*, wages increase rather sharply until around year 2031 with a maximum close to 12% with respect to their 2001 level, and decline slowly after, yet remaining some 3 to 4 % above their initial level. The time profile of wages in essentially identical across professions, as one expects, though differences are far from being negligible, as Figure 16 illustrates. Those time series of wages are in line with those found by MMF2004, which is, to some extent, surprising. Indeed, one would expect that, given the very strong difference in production technologies between the two models – we assume a single sector Cobb-Douglas technology, while they work with an extremely complex multi-level CES structure at the sectoral level – differences in % deviations with respect to the initial level be more important then they are. This apparent robustness of simulation projections is undoubtedly reassuring. The contrast between our results and theirs is, on the other hand, quite drastic when it comes to the interest rate. In both models, and this is as one expects, the interest rate exhibits a

U-shaped time profile between year 2001 and year 2061. But the orders of magnitude are quite different. In our model, as illustrated by Figure 17, the rate of return of capital drops from its initial value around 3.5% to less than 1% thirty years later. It then recovers to some extent and stabilises slightly above 2%. MMF2004, on the other hand, report a rate of return drop that culminates - also thirty years later - 0.8% lower than in 2000: the drop they predict is therefore extremely modest compared to ours. The reason lies certainly to some extent in differences in the two demographic shocks. Indeed, our modelling of the ageing of the Canadian society takes two complementary forms: there is a fall in both the fertility and the mortality rates, respectively, parameters η and γ of equation (19), whereas, MMF2004 do not account for this drift of the life-expectancy. In our model therefore, individuals who expect to die older but to retire from the workforce at the same age 65 adjust their consumption plans and accumulate assets when young at a much more intensive pace. The intensity of the interest rate fall is without doubt due to this supply effect. Note that the two models also differ on their modelling of bequests. In MMF2004, all bequests are voluntary whereas in our formulation there are only involuntary legs. It is however unclear how much this difference affects the position of the aggregate supply function of savings.

6.2 Income inequalities induced by ageing

The general equilibrium factor prices computed using the OLG model are then fed into the microsimulation model to evaluate individual decisions, earnings and income inequalities.

Let us first mention the methodological issue that consists to "animate" this picture in such a way that inequality indices may me computed meaningfully through time. Each of the 62077 existing individuals that are in the data set are moved up the generation ladder making their discrete choices each decade with the new factor prices. A fraction of those individuals are discarded every period from each generation according to the rate $\gamma_{g(k),t}$ of equation (19). At the rate η_t , new generations are entered into the population stock with gender characteristics in identical proportion as in the data set; for these entrants, we randomly generate their preferences using the coefficients estimated on the observed sample; once these new born individuals are endowed with those preferences, they are individually submitted to the choice of education taking into account new valuations of diplomas and then to the work-leisure and profession choices.

Note that in our Canadian data set, and in contrast with the computer-generated data base of Magnani and Mercenier (2006), we have no information on capital rents, but only on labour earnings by individuals. We know from our previous paper, that most income inequalities following an ageing shock will emerge from composition differences between individuals' portfolios in factor endowments. On the other hand, we know from Figures 16 and 17 that most of the action in terms of relative factor prices is between capital and labour, not between different types of labour, given that the price of the latter change rather similarly through time. We therefore do not expect to exhibit very spectacular changes in income inequality indices until we are able to expand the micro data set to include capital earnings, a priority in our future research agenda.

We report in Table 18 the time path of Gini inequality indices by characteristics.

The most remarkable conclusion that seems to emerge from this table is related to women. Though modest for some age groups, the general trend points towards a decline in within age group income inequality among women. The negative trend of the Gini indices is more pronounced as the group becomes older. This is particularly clear – and actually quite spectacular – among older women (aged 55-64) for which the Gini index steadily declines from 0.72272 to 0.67465. Figure 19 highlights this observation: in this graph, we report, for each female age class, an index (=1 at base year 2001) of the Gini index. These curves cannot be compared to one other, of course, but highlight the time path of each inequality indicator separately. The main reason behind this evolution is to be found in participation rates to the labour market. These rates are reported in Tables 20a to 20f. Again, we highlight the case of women aged 55-64, whose participation rate increases steadily during the first five decades, from an initial rate of 46.6% to 55.9%.

For men, no clear trend in participation rates nor in Gini coefficients emerge, except though modestly for older ones, as Figure 21 illustrates.

6.3 Conclusions

This paper is a step further towards linking dynamic general equilibrium models and microsimulation models using rigorous aggregation theory. The approach relies on complex simulation methods as well as on sophisticated statistical estimation techniques. It has been applied on Canadian data in a context of changing population to highlight the potential impact of ageing on income inequalities.

The predictions provided here are tentative since some improvements are still called for. Among the most important, let us mention: (a) The micro data set should be extended to include all earning sources and not only labour earnings. Also, we expect to be able to work on a larger data set with ten-times more individuals (more than 600000). (b) In the discrete-choice for education, a fixed point problem has yet to be solved so that individual agents' asset-valuation of education degrees be consistent with the macro projections. (c) In the OLG model, a more sophisticated production technology should be introduced that acknowledges (as in MMF2004) the imperfectly substitutable nature of individual workers with different characteristics such as age and/or education levels.

No doubt, there is plenty of exciting research to be done in the near future.

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APPENDIX A

Prof1	Senior management occupations (A0)
	Other management occupations (A1, A2, A3)
Prof2	Professional occupations in business and finance (B0)
	Financial, secretarial and administrative occupations (B1, B2, B3)
	Clerical occupations and clerical supervisors (B4, B5)
Prof3	Occupations in natural and applied sciences (C0, C1)
Prof4	Professional occupations in health, registered nurses and supervisors (D0, D1)
	Technical, assisting and related occupations in health (D2, D3)
Prof5	Occupations in social science, government services and religion (E0, E2)
	Teachers and professors (E1)
Prof6	Occupations in art, culture, recreation and sport (F0, F1)
Prof7	Wholesale, technical, insurance, real estate sales specialists, and retail, wholesale and grain buyers (G1)
	Retail trade supervisors, salespersons, sales clerks and cashiers (G2, G3, G011)
	Chefs and cooks, supervisors, and other occupations in food and beverage service (G4, G5, G012)
	Occupations in protective services (G6)
	Childcare and home support workers (G8)
	Service supervisors, occupations in travel and accommodation, attendants in recreation and sport and sales and service occupations, n.e.c. (G7, G9, G013, G014, G015, G016)
Prof8	Contractors and supervisors in trades and transportation (H0)
	Construction trades (H1)
	Other trades occupations (H2, H3, H4, H5)
	Transport and equipment operators (H6, H7)
	Trades helpers, construction, and transportation labourers and related occupations (H8)
Prof9	Occupations unique to primary industries (I0, I1, I2)
Prof10	Supervisors, machine operators and assemblers in manufacturing (J0, J1, J2)
	Labourers in processing, manufacturing and utilities (J3)

Table 1: Types of professions

edu1	None
edu2	Secondary (high) school graduation certificate or equivalent
edu3	Trades certificate or diploma
	Other non-university certificate or diploma
	University certificate or diploma below bachelor level
edu4	Bachelor's degree
edu5	University certificate or diploma above bachelor level
	Master's degree
	Degree in medicine, dentistry, veterinary medicine or optometry
	Earned doctorate

Table 2: Types of education levels

	all individuals	females	males	15-24	25-34	35-44	45-54	55-64
prof1	10.7%	8.1%	13.0%	2.7%	9.0%	12.3%	13.4%	12.3%
prof2	18.4%	28.5%	9.4%	15.1%	19.1%	18.4%	19.3%	18.3%
prof3	6.7%	3.0%	10.0%	5.1%	9.0%	7.3%	5.8%	4.2%
prof4	5.4%	9.3%	2.0%	3.0%	5.7%	5.8%	6.2%	4.6%
prof5	8.2%	11.7%	5.2%	3.9%	9.4%	7.9%	9.9%	7.7%
prof6	2.8%	3.2%	2.4%	3.5%	3.4%	2.4%	2.4%	2.4%
prof7	21.9%	27.2%	17.2%	40.1%	20.7%	18.7%	18.2%	22.2%
prof8	14.9%	2.1%	26.3%	13.8%	13.6%	16.0%	14.9%	16.4%
prof9	3.5%	1.6%	5.2%	4.8%	2.8%	3.2%	3.2%	5.3%
prof10	7.4%	5.2%	9.4%	8.2%	7.4%	8.0%	6.6%	6.6%
total	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
employed	75.9%	71.0%	80.9%	46.5%	89.5%	88.9%	85.9%	58.2%
unemployed	24.1%	29.0%	19.1%	53.5%	10.5%	11.1%	14.1%	41.8%

Table 3: Participation rates by professions and characteristics

	females	males	total	15-24	25-34	35-44	45-54	55-64	total
prof1	35.4%	64.6%	100.0%	2.9%	19.5%	34.1%	31.2%	12.3%	100.0%
prof2	72.8%	27.2%	100.0%	9.7%	24.1%	29.7%	26.0%	10.6%	100.0%
prof3	21.1%	78.9%	100.0%	8.9%	31.0%	32.1%	21.5%	6.6%	100.0%
prof4	80.8%	19.2%	100.0%	6.5%	24.2%	31.6%	28.6%	9.1%	100.0%
prof5	66.7%	33.3%	100.0%	5.5%	26.2%	28.5%	29.8%	10.0%	100.0%
prof6	53.9%	46.1%	100.0%	14.8%	28.3%	26.2%	21.5%	9.3%	100.0%
prof7	58.3%	41.7%	100.0%	21.5%	21.8%	25.2%	20.7%	10.8%	100.0%
prof8	6.7%	93.3%	100.0%	10.9%	21.1%	31.7%	24.7%	11.7%	100.0%
prof9	21.6%	78.4%	100.0%	16.0%	18.4%	27.0%	22.7%	15.9%	100.0%
prof10	32.8%	67.2%	100.0%	13.1%	22.9%	32.2%	22.3%	9.5%	100.0%

Table 4: Participation rates by professions and characteristics

	edu1	edu2	edu3	edu4	edu5
prof1	7.0%	10.0%	10.1%	16.1%	16.9%
prof2	11.8%	23.1%	20.2%	18.4%	11.5%
prof3	1.3%	3.4%	8.2%	13.8%	13.9%
prof4	1.4%	1.8%	8.5%	7.1%	12.1%
prof5	1.6%	2.5%	5.3%	23.5%	35.0%
prof6	1.2%	2.3%	3.1%	5.1%	3.5%
prof7	30.0%	28.7%	19.7%	11.0%	5.2%
prof8	23.8%	14.6%	17.4%	2.3%	1.2%
prof9	7.6%	4.0%	2.3%	1.0%	0.0%
prof10	14.3%	9.6%	5.1%	1.8%	0.9%
total	100.0%	100.0%	100.0%	100.0%	100.0%
employed	69.5%	80.8%	79.0%	72.9%	71.9%
unemployed	30.5%	19.2%	21.0%	27.1%	28.1%

Table 5: Participation rates by professions and characteristics

	edu1	edu2	edu3	edu4	edu5	total
prof1	13.5%	23.2%	32.5%	20.2%	10.6%	100.0%
prof2	13.2%	31.3%	37.9%	13.4%	4.2%	100.0%
prof3	4.1%	12.4%	42.2%	27.5%	13.8%	100.0%
prof4	5.2%	8.4%	54.0%	17.4%	15.0%	100.0%
prof5	3.9%	7.6%	22.0%	38.0%	28.4%	100.0%
prof6	8.7%	20.4%	38.1%	24.4%	8.4%	100.0%
prof7	28.2%	32.5%	31.0%	6.7%	1.6%	100.0%
prof8	32.8%	24.4%	40.2%	2.1%	0.5%	100.0%
prof9	44.8%	28.5%	22.8%	3.8%	0.0%	100.0%
prof10	39.9%	32.3%	23.8%	3.3%	0.8%	100.0%

Table 6: Participation rates by professions and characteristics

	constant	age	age2	sex	edu
prof1	7.97219***	0.75318***	-0.09908***	0.41118***	0.20208***
	(0.11826)	(0.06984)	(0.01056)	(0.02637)	(0.01071)
prof2	8.06459***	0.76908***	-0.10322***	0.36427***	0.09432***
	(0.07161)	(0.04293)	(0.00698)	(0.02228)	(0.00991)
prof3	8.06655***	0.88074***	-0.11434***	0.31069***	0.11805***
	(0.13298)	(0.07366)	(0.01225)	(0.03890)	(0.01590)
prof4	7.74234***	0.68394***	-0.08494***	0.34172***	0.26561***
	(0.15339)	(0.09424)	(0.01510)	(0.05063)	(0.02019)
prof5	7.33750***	0.87950***	-0.11228***	0.28842***	0.25084***
	(0.12626)	(0.07701)	(0.01218)	(0.03315)	(0.01482)
prof6	7.70326***	0.90141***	-0.13685***	0.34617***	0.10411***
	(0.19730)	(0.12401)	(0.02077)	(0.06091)	(0.02903)
prof7	7.56739***	0.58579***	-0.07997***	0.56974***	0.12074***
	(0.06409)	(0.04109)	(0.00705)	(0.02238)	(0.01141)
prof8	7.47340***	0.68293***	-0.09174***	0.65737***	0.12100***
	(0.11621)	(0.04861)	(0.00791)	(0.04704)	(0.01285)
prof9	7.7700***	0.62175***	-0.09925***	0.571480***	0.06429**
	(0.18663)	(0.09891)	(0.01619)	(0.06383)	(0.02943)
prof10	8.00813***	0.47433***	-0.05549***	0.62752***	0.03592*
	(0.11984)	(0.06972)	(0.01165)	(0.03677)	(0.01908)

Note: ***, ** and * represent 1%, 5% and 10% significance level

Table 7: Estimation of the Mincer equation

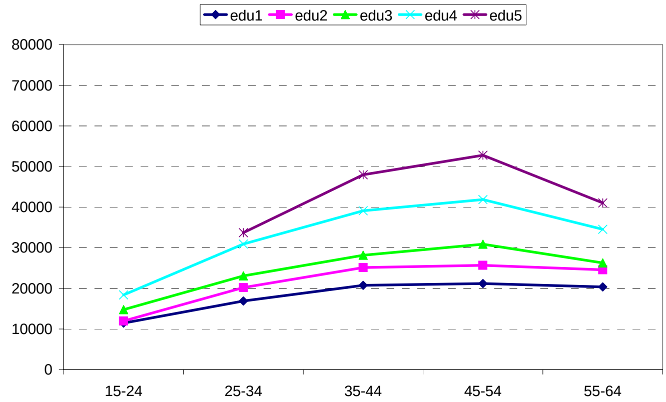

Figure 8a: Earning profile (females)

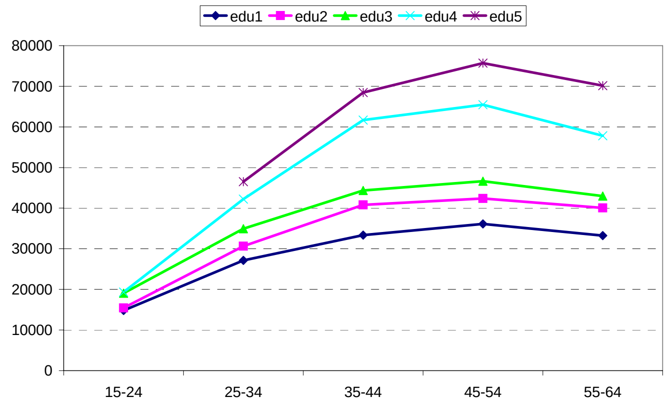

Figure 8b: Earning profile (males)

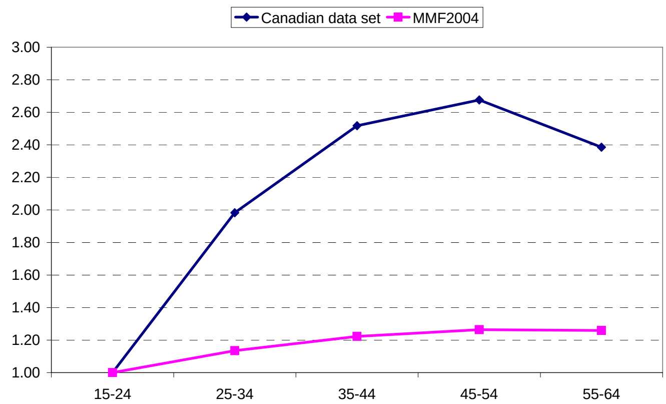

Figure 9: Comparison of earning profiles

	edu1	edu2	edu3	edu4	edu5
15-24	1.23694***	1.23419***	0.92615***	1.21691***	
	(0.16735)	(0.10061)	(0.07677)	(0.17441)	
25-34	0.58276***	0.59806***	0.67282***	0.85371***	0.71045***
	(0.06392)	(0.05004)	(0.03920)	(0.05665)	(0.08149)
35-44	0.63548***	0.73388***	0.66839***	0.79949***	0.92532***
	(0.05272)	(0.04740)	(0.03481)	(0.06237)	(0.10349)
45-54	0.60441***	0.66756***	0.76359***	0.79713***	0.74252***
	(0.05067)	(0.04850)	(0.04437)	(0.07449)	(0.10543)
55-64	0.62516***	0.77828***	0.72940***	0.58381***	0.50046***
	(0.06519)	(0.08602)	(0.06742)	(0.12400)	(0.13400)

Note: ***, ** and * represent 1%, 5% and 10% significance level

Table 10a: Logit estimation for the choice of the profession - Females

	edu1	edu2	edu3	edu4	edu5
15-24	1.00625***	0.97040***	0.88441***	0.93376***	
	(0.07429)	(0.05511)	(0.07411)	(0.16729)	
25-34	0.57879***	0.77753***	0.78002***	0.82156***	0.71824***
	(0.05371)	(0.05356)	(0.04435)	(0.06160)	(0.08970)
35-44	0.62534***	0.87370***	0.83677***	0.80395***	0.71223***
	(0.04352)	(0.05652)	(0.03924)	(0.06678)	(0.07928)
45-54	0.72736***	0.71176***	0.78935***	1.01928***	0.91391***
	(0.04993)	(0.05000)	(0.04302)	(0.08333)	(0.10124)
55-64	0.67467***	0.78480***	0.70220***	0.59265***	0.52091***
	(0.05562)	(0.08543)	(0.05888)	(0.09440)	(0.08775)

Note: ***, ** and * represent 1%, 5% and 10% significance level

Table 10b: Logit estimation for the choice of the profession - Males

	Females	Males
edu1	-0.141	-0.138
edu2	-0.184	-0.113
edu3	-0.159	-0.137
edu4	-0.207	-0.105
edu5	-0.161	-0.123

Table 11: Estimated wage penalty of the unemployed

	15-24
β_2	1.11261***
	(0.24765)

Note: significant at 1% level

Table 12: Estimates of choice of education

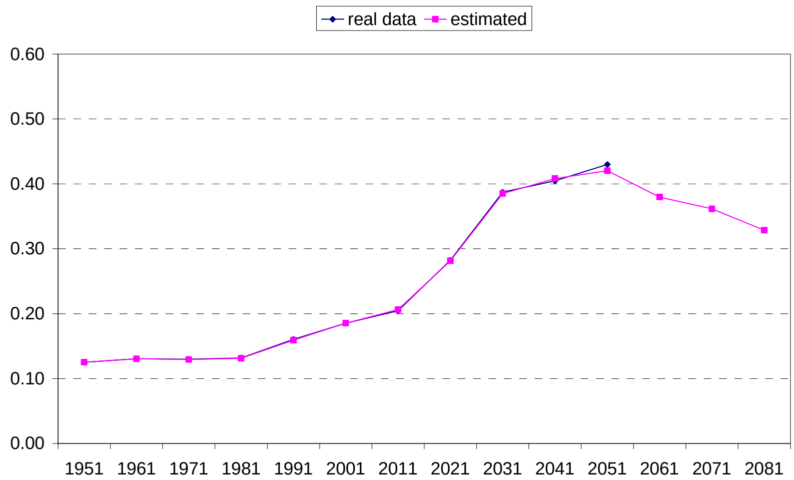

Figure 13: Old-age dependency ratio

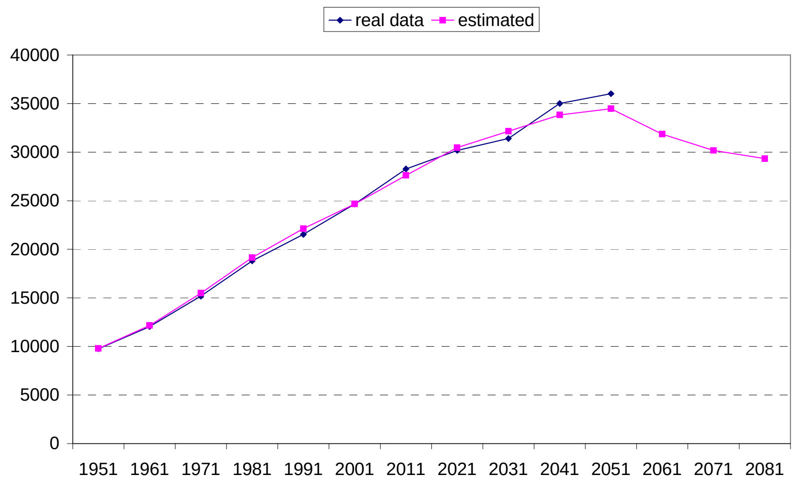

Figure 14: Total population

	2001	2011	2021	2031	2041	2051
Total population	1.000	1.119	1.235	1.304	1.371	1.397
Number of workers	1.000	1.053	1.083	1.099	1.127	1.144
Per unit of effective labour	1.000	1.048	1.057	1.081	1.125	1.142
GDP	1.000	1.094	1.153	1.199	1.247	1.261
Per capita GDP	1.000	0.977	0.934	0.920	0.909	0.902
Per worker GDP	1.000	1.039	1.065	1.092	1.106	1.102
Per unit of effective labour GDP	1.000	1.043	1.091	1.109	1.108	1.104
Rate of growth of GDP		9.4%	5.4%	4.0%	4.0%	1.1%
Rate of growth of per capita GDP		-2.3%	-4.4%	-1.5%	-1.1%	-0.8%
Rate of growth of per worker GDP		3.9%	2.6%	2.5%	1.3%	-0.4%
Rate of growth of per unit of effective labour GDP		4.3%	4.6%	1.6%	0.0%	-0.4%
Investements / GDP	1.000	1.028	0.984	0.957	0.894	0.583
Average wage rate	1.000	1.046	1.090	1.107	1.109	1.107
Interest rate	1.000	0.681	0.383	0.289	0.289	0.311

Table 15: Macro results

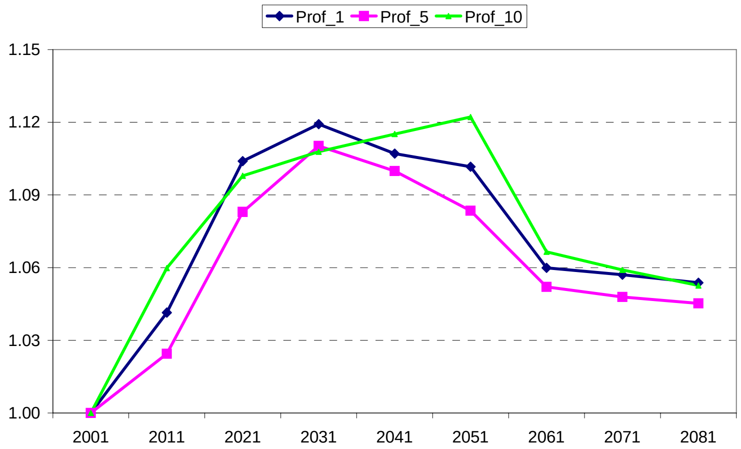

Figure 16: Evolution of the per unit of effective labour wages for three profession types

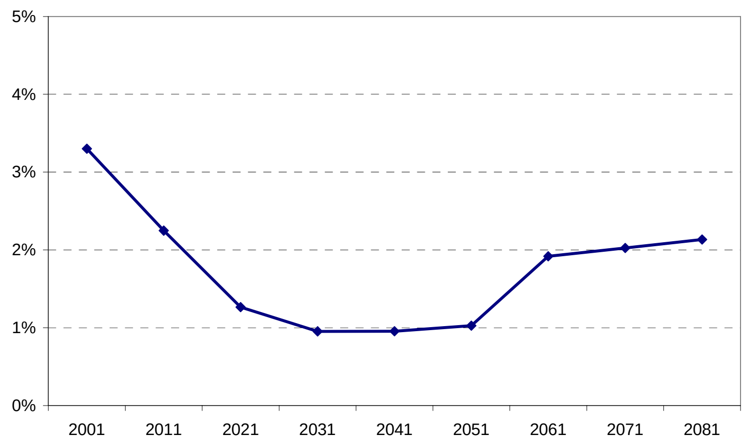

Figure 17: Evolution of the interest rate

	2001	2011	2021	2031	2041	2051
all individuals	0.55871	0.56532	0.55846	0.54773	0.55502	0.55293
15-24	0.70914	0.70173	0.69779	0.69983	0.69895	0.69687
25-34	0.44803	0.44421	0.44226	0.44856	0.44073	0.44413
35-44	0.45847	0.45419	0.44786	0.44761	0.44971	0.45049
45-54	0.47860	0.47115	0.46500	0.46004	0.45872	0.46046
55-64	0.67354	0.66103	0.65484	0.64587	0.64501	0.63526
workers	0.41880	0.41850	0.41666	0.41626	0.41606	0.41410
females - 15-24	0.71704	0.71548	0.70564	0.71033	0.70677	0.70602
females - 25-34	0.47653	0.47957	0.47802	0.48322	0.47725	0.47429
females- 35-44	0.49064	0.48735	0.48272	0.47461	0.48165	0.48172
females- 45-54	0.50598	0.49352	0.48681	0.47601	0.47674	0.48335
females - 55-64	0.72272	0.70012	0.69373	0.68521	0.67686	0.67465
females	0.58143	0.58857	0.58246	0.56910	0.57598	0.57697
males - 15-24	0.69739	0.68618	0.68769	0.68675	0.68791	0.68376
males 25-34	0.40187	0.39829	0.39779	0.40265	0.39612	0.39905
males 35-44	0.40110	0.39616	0.39113	0.39529	0.39455	0.39585
males 45-54	0.42303	0.42096	0.41586	0.41680	0.41766	0.41472
males 55-64	0.60181	0.59518	0.59204	0.58269	0.59158	0.57313
males	0.51525	0.52298	0.51703	0.50768	0.51768	0.51120

Table 18: Evolution of the Gini index

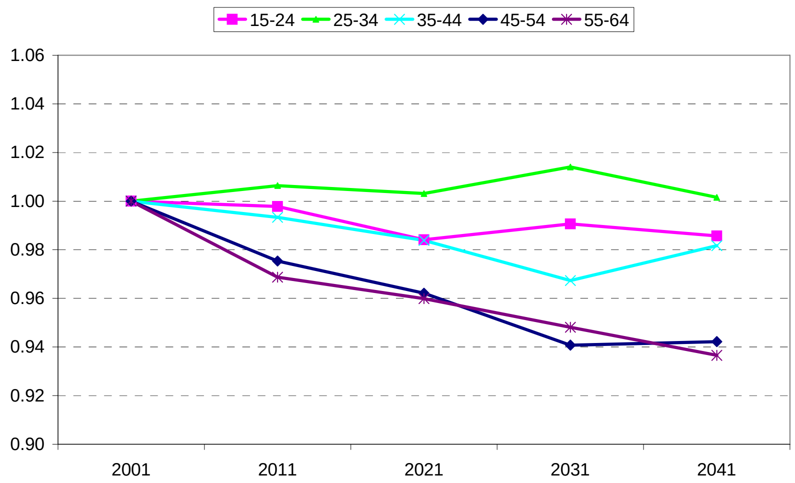

Figure 19: Evolution of the Gini index for females

	Females					Males					Total				
	15-24	25-34	35-44	45-54	55-64	15-24	25-34	35-44	45-54	55-64	15-24	25-34	35-44	45-54	55-64
prof1 / workers	3.3%	7.4%	9.4%	11.0%	8.0%	3.2%	10.5%	15.0%	15.7%	14.6%	3.2%	9.0%	12.3%	13.5%	11.9%
prof2 / workers	22.9%	26.1%	27.9%	28.3%	29.8%	9.1%	10.6%	8.8%	9.9%	8.9%	15.4%	18.2%	17.9%	18.5%	17.6%
prof3 / workers	2.5%	4.8%	3.7%	1.9%	0.8%	7.9%	13.2%	9.9%	9.2%	7.1%	5.4%	9.1%	6.9%	5.8%	4.5%
prof4 / workers	7.3%	10.4%	10.4%	10.7%	8.4%	1.0%	1.8%	2.6%	2.6%	1.9%	3.9%	6.0%	6.3%	6.4%	4.6%
prof5 / workers	6.7%	13.2%	11.0%	12.7%	11.3%	2.2%	5.6%	4.7%	6.9%	6.5%	4.3%	9.3%	7.7%	9.6%	8.5%
prof6 / workers	5.5%	3.8%	3.1%	2.7%	2.6%	3.2%	3.5%	2.1%	2.4%	2.0%	4.2%	3.7%	2.6%	2.5%	2.3%
prof7 / workers	41.0%	25.1%	24.0%	23.3%	28.8%	29.4%	17.0%	14.6%	14.8%	18.3%	34.7%	21.0%	19.1%	18.8%	22.7%
prof8 / workers	2.7%	2.4%	2.2%	2.4%	2.2%	23.1%	23.2%	26.8%	24.9%	26.2%	13.7%	13.1%	15.1%	14.3%	16.1%
prof9 / workers	2.4%	1.4%	2.2%	1.8%	3.0%	8.0%	5.0%	5.1%	4.9%	7.1%	5.4%	3.3%	3.8%	3.5%	5.4%
prof10 / workers	5.8%	5.3%	6.0%	5.1%	5.1%	12.9%	9.6%	10.5%	8.8%	7.4%	9.6%	7.5%	8.4%	7.1%	6.4%
	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
workers / active population	43.1%	85.8%	84.6%	80.9%	46.6%	49.9%	94.3%	94.3%	91.2%	66.7%	46.5%	90.0%	89.4%	86.1%	56.5%

Table 20a: Participation rates by groups - year 2001

	Females					Males					Total				
	15-24	25-34	35-44	45-54	55-64	15-24	25-34	35-44	45-54	55-64	15-24	25-34	35-44	45-54	55-64
prof1 / workers	3.6%	8.3%	9.8%	10.5%	7.8%	2.7%	10.4%	15.0%	15.9%	16.5%	3.1%	9.4%	12.5%	13.3%	12.9%
prof2 / workers	22.8%	26.9%	28.7%	29.3%	32.1%	9.1%	10.7%	9.2%	10.3%	10.0%	15.3%	18.4%	18.7%	19.3%	19.1%
prof3 / workers	2.8%	5.5%	4.4%	2.2%	0.8%	8.2%	13.3%	11.2%	9.2%	7.9%	5.8%	9.6%	7.9%	5.9%	5.0%
prof4 / workers	6.1%	10.1%	10.6%	11.1%	9.3%	1.3%	1.9%	2.7%	2.5%	2.1%	3.5%	5.8%	6.5%	6.6%	5.1%
prof5 / workers	6.9%	13.8%	12.7%	13.2%	12.2%	1.8%	5.4%	4.8%	6.1%	7.1%	4.1%	9.4%	8.6%	9.5%	9.2%
prof6 / workers	5.9%	4.3%	3.6%	3.1%	3.3%	3.4%	2.9%	2.2%	2.4%	2.6%	4.5%	3.5%	2.9%	2.7%	2.9%
prof7 / workers	41.5%	22.7%	20.9%	22.3%	25.7%	29.4%	17.1%	14.8%	15.1%	18.9%	34.9%	19.8%	17.8%	18.5%	21.7%
prof8 / workers	2.7%	2.1%	2.6%	2.1%	1.8%	24.4%	22.8%	25.1%	24.8%	22.1%	14.5%	13.0%	14.2%	14.0%	13.7%
prof9 / workers	2.3%	1.1%	1.7%	1.3%	2.7%	8.0%	4.5%	4.3%	5.4%	6.4%	5.4%	2.9%	3.0%	3.4%	4.9%
prof10 / workers	5.4%	5.1%	5.1%	5.0%	4.2%	11.8%	11.1%	10.6%	8.2%	6.5%	8.9%	8.2%	8.0%	6.7%	5.6%
	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
workers / active population	43.8%	86.8%	85.2%	83.1%	49.4%	51.5%	93.9%	94.8%	92.3%	70.2%	47.7%	90.4%	89.9%	87.7%	59.8%

Table 20b: Participation rates by groups - year 2011

	Females					Males					Total				
	15-24	25-34	35-44	45-54	55-64	15-24	25-34	35-44	45-54	55-64	15-24	25-34	35-44	45-54	55-64
prof1 / workers	4.0%	8.1%	9.0%	10.9%	8.6%	2.6%	10.1%	16.7%	16.6%	16.8%	3.2%	9.2%	13.0%	13.8%	13.3%
prof2 / workers	21.9%	26.6%	28.9%	26.5%	31.1%	9.4%	11.4%	9.0%	9.7%	10.3%	15.1%	18.6%	18.5%	17.9%	19.2%
prof3 / workers	2.3%	5.1%	4.7%	2.9%	1.1%	7.6%	12.3%	10.9%	9.7%	7.3%	5.2%	8.9%	7.9%	6.4%	4.6%
prof4 / workers	6.3%	10.5%	11.9%	12.6%	10.9%	0.8%	1.9%	2.7%	2.7%	1.5%	3.3%	6.0%	7.1%	7.6%	5.5%
prof5 / workers	6.0%	14.7%	13.5%	16.4%	14.1%	2.3%	5.5%	5.0%	7.7%	7.0%	4.0%	9.9%	9.0%	12.0%	10.0%
prof6 / workers	5.8%	4.2%	3.5%	3.4%	3.2%	3.6%	3.3%	2.4%	2.7%	2.5%	4.6%	3.7%	2.9%	3.0%	2.8%
prof7 / workers	43.0%	23.5%	20.5%	19.5%	23.8%	28.3%	17.3%	14.4%	14.4%	17.7%	35.1%	20.2%	17.3%	16.9%	20.4%
prof8 / workers	2.2%	1.9%	2.0%	1.9%	1.4%	22.9%	22.9%	23.9%	23.2%	24.0%	13.4%	12.9%	13.5%	12.8%	14.3%
prof9 / workers	3.0%	1.2%	1.7%	1.4%	2.3%	9.2%	4.8%	4.7%	5.3%	6.0%	6.4%	3.1%	3.3%	3.4%	4.4%
prof10 / workers	5.4%	4.3%	4.3%	4.4%	3.6%	13.3%	10.5%	10.2%	8.0%	6.8%	9.7%	7.6%	7.4%	6.2%	5.4%
	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
workers / active population	44.7%	87.5%	87.6%	84.5%	52.4%	51.5%	94.5%	94.7%	92.2%	70.0%	48.1%	91.0%	91.2%	88.3%	61.1%

Table 20c: Participation rates by groups - year 2021

	Females					Males					Total				
	15-24	25-34	35-44	45-54	55-64	15-24	25-34	35-44	45-54	55-64	15-24	25-34	35-44	45-54	55-64
prof1 / workers	4.3%	8.1%	9.7%	10.5%	8.9%	2.8%	10.9%	15.9%	17.1%	17.3%	3.5%	9.6%	12.9%	14.0%	13.6%
prof2 / workers	22.0%	25.6%	27.9%	27.8%	29.9%	10.1%	10.9%	9.1%	10.5%	11.4%	15.6%	17.8%	18.1%	18.8%	19.6%
prof3 / workers	2.8%	4.7%	4.4%	2.6%	0.8%	7.1%	12.4%	11.2%	9.4%	7.6%	5.1%	8.7%	8.0%	6.1%	4.6%
prof4 / workers	6.8%	10.7%	11.6%	12.2%	11.3%	1.0%	2.3%	2.6%	2.6%	1.7%	3.7%	6.3%	6.8%	7.2%	6.0%
prof5 / workers	7.1%	14.6%	14.2%	17.2%	16.8%	2.3%	5.5%	5.1%	8.0%	7.8%	4.5%	9.8%	9.4%	12.4%	11.8%
prof6 / workers	4.9%	3.5%	3.4%	3.7%	4.1%	3.8%	3.1%	2.6%	2.7%	2.5%	4.3%	3.3%	3.0%	3.2%	3.2%
prof7 / workers	40.3%	24.3%	20.9%	19.7%	21.8%	29.8%	16.9%	15.0%	13.9%	17.8%	34.6%	20.4%	17.8%	16.6%	19.5%
prof8 / workers	2.9%	2.1%	2.0%	1.4%	1.3%	21.5%	24.0%	24.9%	23.0%	21.8%	12.9%	13.6%	14.0%	12.7%	12.7%
prof9 / workers	2.3%	1.5%	1.3%	1.2%	1.7%	8.9%	4.1%	4.2%	5.2%	5.7%	5.8%	2.8%	2.8%	3.3%	3.9%
prof10 / workers	6.6%	5.0%	4.6%	3.7%	3.2%	12.7%	9.9%	9.5%	7.5%	6.5%	9.9%	7.6%	7.1%	5.7%	5.1%
	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
workers / active population	44.5%	86.6%	88.1%	86.0%	54.6%	51.2%	95.2%	95.3%	92.0%	71.5%	47.9%	90.9%	91.7%	89.0%	62.9%

Table 20d: Participation rates by groups - year 2031

	Females					Males					Total				
	15-24	25-34	35-44	45-54	55-64	15-24	25-34	35-44	45-54	55-64	15-24	25-34	35-44	45-54	55-64
prof1 / workers	3.4%	8.0%	9.6%	10.8%	8.1%	2.5%	10.8%	16.0%	16.2%	17.5%	2.9%	9.5%	13.0%	13.6%	13.3%
prof2 / workers	22.3%	26.4%	27.8%	27.5%	30.4%	10.3%	11.1%	8.8%	10.7%	11.9%	15.8%	18.4%	17.8%	18.7%	20.0%
prof3 / workers	2.5%	5.2%	4.5%	2.7%	1.1%	6.9%	12.7%	11.1%	9.7%	8.5%	4.9%	9.1%	8.0%	6.3%	5.2%
prof4 / workers	6.2%	10.9%	11.5%	12.4%	11.3%	1.1%	2.2%	2.6%	2.7%	1.6%	3.4%	6.3%	6.8%	7.3%	5.9%
prof5 / workers	6.7%	13.9%	13.1%	17.4%	16.9%	2.3%	5.3%	5.0%	7.0%	7.2%	4.3%	9.4%	8.9%	12.0%	11.5%
prof6 / workers	6.6%	4.5%	3.5%	3.5%	4.5%	3.4%	3.7%	2.0%	2.6%	2.8%	4.9%	4.1%	2.7%	3.1%	3.5%
prof7 / workers	41.2%	23.8%	21.8%	19.3%	22.2%	30.0%	17.5%	14.9%	15.4%	16.9%	35.2%	20.5%	18.1%	17.3%	19.2%
prof8 / workers	2.5%	2.1%	2.0%	1.6%	1.0%	23.6%	21.8%	25.0%	23.2%	21.3%	13.9%	12.4%	14.1%	12.8%	12.3%
prof9 / workers	2.8%	1.2%	1.6%	1.5%	1.5%	8.0%	5.0%	4.7%	4.8%	6.3%	5.6%	3.2%	3.3%	3.2%	4.2%
prof10 / workers	5.9%	4.1%	4.6%	3.3%	2.9%	11.8%	10.0%	9.9%	7.6%	6.1%	9.1%	7.2%	7.4%	5.5%	4.7%
	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
workers / active population	44.7%	87.8%	87.3%	86.6%	55.9%	51.5%	94.0%	95.4%	92.4%	69.4%	48.1%	90.9%	91.4%	89.5%	62.7%

Table 20e: Participation rates by groups - year 2041

	Females					Males					Total				
	15-24	25-34	35-44	45-54	55-64	15-24	25-34	35-44	45-54	55-64	15-24	25-34	35-44	45-54	55-64
prof1 / workers	3.4%	7.7%	9.2%	10.9%	8.2%	3.4%	10.9%	15.8%	15.8%	17.4%	3.4%	9.3%	12.7%	13.5%	13.5%
prof2 / workers	22.0%	26.7%	28.9%	27.7%	30.4%	9.1%	11.1%	9.6%	11.2%	11.0%	15.1%	18.5%	18.7%	19.1%	19.3%
prof3 / workers	2.3%	5.2%	4.1%	2.8%	1.2%	6.6%	12.7%	10.7%	8.8%	8.4%	4.6%	9.2%	7.6%	6.0%	5.3%
prof4 / workers	6.0%	9.6%	11.2%	12.6%	12.9%	0.8%	2.2%	2.7%	2.6%	1.8%	3.2%	5.7%	6.7%	7.4%	6.6%
prof5 / workers	6.2%	13.7%	13.6%	16.9%	17.0%	2.1%	5.3%	4.8%	7.7%	7.5%	4.0%	9.3%	8.9%	12.1%	11.6%
prof6 / workers	5.1%	4.4%	3.6%	3.6%	4.5%	3.8%	3.4%	2.4%	2.7%	2.7%	4.4%	3.9%	3.0%	3.2%	3.5%
prof7 / workers	43.2%	24.4%	21.1%	18.6%	20.3%	28.3%	17.2%	14.7%	14.4%	17.2%	35.2%	20.6%	17.7%	16.4%	18.5%
prof8 / workers	2.7%	2.2%	2.3%	1.7%	0.9%	22.5%	22.5%	24.9%	23.2%	22.2%	13.3%	12.8%	14.2%	13.0%	13.1%
prof9 / workers	2.9%	1.2%	1.4%	1.4%	1.8%	9.5%	4.1%	4.2%	5.5%	5.4%	6.4%	2.7%	2.9%	3.5%	3.8%
prof10 / workers	6.1%	4.9%	4.7%	3.7%	2.8%	13.9%	10.5%	10.0%	8.0%	6.4%	10.3%	7.9%	7.5%	6.0%	4.9%
	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
workers / active population	45.4%	87.3%	86.6%	85.5%	54.8%	51.5%	94.5%	95.2%	92.6%	72.0%	48.5%	90.9%	91.0%	89.1%	63.5%

Table 20f: Participation rates by groups - year 2051

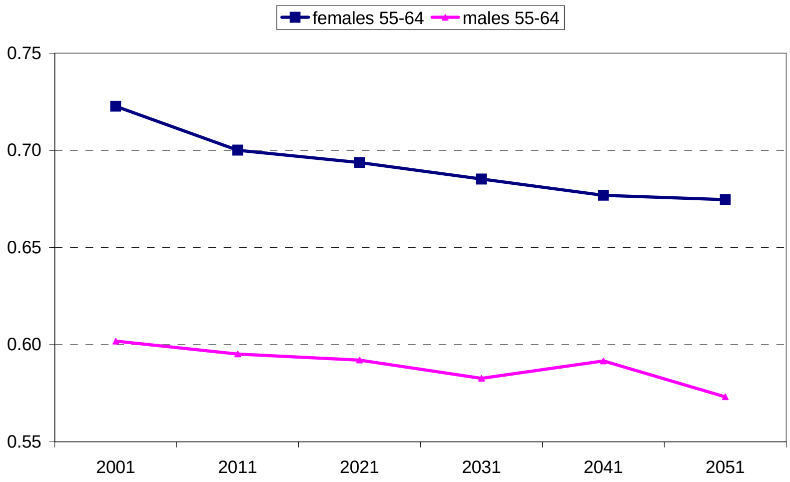

Figure 21: Evolution of the Gini index for females and males aged 55-64

APPENDIX B

Some statistics about labour participation by age, sex and education level, computed from the micro data set.

	edu1	edu2	edu3	edu4	edu5
prof1	8	32	21	15	0
prof2	48	242	214	69	0
prof3	0	15	21	25	0
prof4	10	26	87	18	0
prof5	8	34	62	53	0
prof6	12	42	39	17	0
prof7	287	612	322	70	0
prof8	12	16	16	0	0
prof9	21	17	7	0	0
prof10	21	63	26	0	0
workers	427	1099	815	267	0
unemployed	175	434	1323	900	474
total	602	1533	2138	1167	474

Table 22: Females, 15-24

	edu1	edu2	edu3	edu4	edu5
prof1	17	30	16	9	0
prof2	57	129	59	20	0
prof3	13	71	92	44	0
prof4	5	10	10	0	0
prof5	5	15	11	27	0
prof6	12	39	26	6	0
prof7	296	443	166	31	0
prof8	209	305	199	8	0
prof9	89	110	20	0	0
prof10	124	165	59	0	0
workers	827	1317	658	145	0
unemployed	338	443	1036	843	418
total	1165	1760	1694	988	418

Table 23: Males, 15-24

	edu1	edu2	edu3	edu4	edu5
prof1	26	78	151	104	39
prof2	118	330	684	280	75
prof3	0	25	80	102	41
prof4	11	36	281	130	59
prof5	22	39	182	326	162
prof6	9	18	79	77	17
prof7	303	347	487	147	28
prof8	38	29	39	0	0
prof9	22	21	13	8	0
prof10	82	88	73	12	0
workers	631	1011	2069	1186	421
unemployed	301	246	223	103	35
total	932	1257	2292	1289	456

Table 24: Females, 25-34

	edu1	edu2	edu3	edu4	edu5
prof1	63	157	160	154	51
prof2	68	125	191	162	52
prof3	17	76	290	251	99
prof4	0	17	29	17	39
prof5	6	16	31	146	89
prof6	8	40	69	41	12
prof7	177	292	338	113	21
prof8	349	371	619	32	5
prof9	101	68	60	10	0
prof10	180	172	153	35	6
workers	969	1334	1940	961	374
unemployed	157	97	57	39	27
total	1126	1431	1997	1000	401

Table 25: Males, 25-34

	edu1	edu2	edu3	edu4	edu5
prof1	62	154	210	100	63
prof2	228	687	811	203	44
prof3	10	24	107	63	33
prof4	25	42	376	125	77
prof5	36	70	206	280	175
prof6	9	25	76	64	14
prof7	410	471	560	88	28
prof8	50	48	48	10	0
prof9	37	35	40	0	0
prof10	168	116	81	10	5
workers	1035	1672	2515	943	439
unemployed	448	344	306	119	60
total	1483	2016	2821	1062	499

Table 26: Females, 35-44

	edu1	edu2	edu3	edu4	edu5
prof1	153	215	375	256	128
prof2	87	145	184	138	48
prof3	34	77	361	182	125
prof4	7	18	59	27	51
prof5	12	16	57	128	127
prof6	20	22	64	27	21
prof7	261	281	390	87	32
prof8	649	462	888	57	17
prof9	152	81	87	14	0
prof10	262	229	214	31	8
workers	1637	1546	2679	947	557
unemployed	214	109	106	27	19
total	1851	1655	2785	974	576

Table 27: Males, 35-44

	edu1	edu2	edu3	edu4	edu5
prof1	84	156	162	88	64
prof2	247	598	639	103	45
prof3	5	23	36	31	15
prof4	45	41	369	88	36
prof5	38	57	169	246	212
prof6	10	32	59	33	15
prof7	440	404	341	53	17
prof8	60	31	27	0	0
prof9	32	25	20	5	0
prof10	151	83	42	6	0
workers	1112	1450	1864	653	404
unemployed	577	345	282	61	34
total	1689	1795	2146	714	438

Table 28: Females, 45-54

	edu1	edu2	edu3	edu4	edu5
prof1	120	231	327	212	124
prof2	95	178	188	113	49
prof3	33	59	261	136	83
prof4	7	14	49	17	64
prof5	10	20	70	158	179
prof6	16	32	45	26	13
prof7	241	257	290	70	22
prof8	536	351	701	27	8
prof9	127	72	78	15	0
prof10	200	146	126	16	8
workers	1385	1360	2135	790	550
unemployed	298	130	133	47	17
total	1683	1490	2268	837	567

Table 29: Males, 45-54

	edu1	edu2	edu3	edu4	edu5
prof1	45	36	58	13	14
prof2	147	211	230	40	18
prof3	0	0	7	0	6
prof4	23	10	110	23	15
prof5	14	21	53	58	71
prof6	7	7	24	13	7
prof7	299	132	163	15	5
prof8	31	7	7	0	0
prof9	25	16	13	0	0
prof10	79	31	8	0	0
workers	670	471	673	162	136
unemployed	1136	438	512	117	54
total	1806	909	1185	279	190

Table 30: Females, 55-64

	edu1	edu2	edu3	edu4	edu5
prof1	103	79	156	65	49
prof2	53	70	84	31	31
prof3	17	23	83	38	35
prof4	0	0	10	0	41
prof5	0	8	16	57	89
prof6	11	9	17	15	11
prof7	198	125	146	18	11
prof8	373	96	288	11	7
prof9	134	26	38	11	0
prof10	126	34	49	5	0
workers	1015	470	887	251	274
unemployed	626	201	337	86	92
total	1641	671	1224	337	366

Table 31: Males, 55-64

Conclusion Générale

L'objectif de cette thèse est de quantifier les impacts économiques du vieillissement démographique à l'aide d'une approche d'équilibre général.

Les modèles d'équilibre général appliqué, grâce à leurs fondements théoriques, représentent un outil indispensable dans l'évaluation des impacts de différentes politiques économiques ou d'autres chocs exogènes. En particulier, les modèles OLG appliqués introduits par Auerbach et Kotlikoff (1987), représentent un outil approprié dans l'analyse des conséquences du vieillissement démographique du fait qu'ils permettent de différencier le comportement des générations, en particulier en termes de participation au marché du travail et en termes de consommation et d'épargne.

La thèse, présentée sous forme de 4 papiers, est composée de deux parties.

La première partie est composée de deux articles concernant l'analyse des conséquences du vieillissement en Italie où le phénomène démographique est parmi les plus préoccupants au monde à cause des niveaux très faibles de la fécondité ; le cas Italien représente un cas d'étude intéressant aussi pour le fait que l'Italie, à partir des années 90, à mis en place plusieurs réformes du système de retraite afin de faire face à une évolution financière du système de retraite qui autrement aurait été catastrophique dans les décennies futures. En particulier, les réformes introduites en 1992 (réforme Dini) et 1995 (réforme Amato) ont modifié la règle de calcul des pensions en introduisant une méthode où les pensions sont liées aux cotisations sociales versées par les travailleurs pendant toute la vie active et une règle d'indexation des pensions sur la base de l'inflation. De plus, en 2004, le gouvernement

Berlusconi a introduit une nouvelle réforme qui augmente l'âge minimum de départ à la retraite.

L'objectif de la première partie de la thèse est d'analyser les effets macroéconomiques et, en particulier, les effets sur le système de retraite, du vieillissement démographique et des réformes récemment introduites. Le modèle utilisé dans cette analyse est un modèle OLG appliqué du type Auerbach-Kotlikoff (1987) comprenant 15 générations (15-24, 25-34, ..., 90-94). Par rapport au modèle standard Auerbach-Kotlikoff, les principales innovations concernent l'introduction d'un mécanisme de croissance endogène à la Lucas (1988), de la mortalité et de l'immigration. De plus, nous avons considéré une offre de travail endogène et des héritages volontaires pour ceux qui meurent à la dernière période et des héritages involontaires en cas de mort prématurée.

Le mécanisme de croissance endogène est introduit en considérant l'investissement en capital humain effectué par les individus âgés de 20-24. L'introduction de l'investissement en capital humain apparaît important dans un contexte de vieillissement de la population qui, à travers une augmentation du ratio capital - travail, provoquera une augmentation des salaires et une baisse des taux d'intérêt, ce qui normalement incitera les jeunes à consacrer plus de temps dans l'éducation. De plus, une augmentation de l'investissement en l'éducation aura un impact positif sur la croissance économique et, par conséquent, sur la situation financière du système de retraite.

L'introduction de la mortalité et de l'immigration nous a permis de reproduire l'évolution démographique d'une façon très précise. L'introduction de l'immigration nous a permis aussi de simuler, dans le deuxième papier, les effets de différentes politiques migratoires.

Ce modèle est utilisé dans les deux premiers papiers pour simuler les impacts de différents chocs économiques. Dans le premier papier, nous analysons les effets sur le système macroéconomique Italien, et en particulier sur le système de retraite, de l'introduction des réformes Amato et Dini qui prévoient l'indexation des pensions à l'inflation et la modification de la règle de calcul des retraites. L'objectif de ce papier est d'analyser l'efficacité de ces réformes. Pour cette raison nous avons comparé les résultats des simulations qui considèrent l'introduction des réformes Amato et Dini avec ceux obtenus en supposant que ces réformes n'avaient pas été introduites. Cette comparaison montre que les réformes étaient absolument indispensables pour réduire les déficits système de retraite qui autrement auraient été énormes. Cependant, il est aussi possible d'affirmer que ces réformes ne sont pas suffisantes, ni à court terme, ni à long terme, à cause des déficits importants prévus (de l'ordre de 3 - 5% par rapport au PIB). Pour faire face à cette situation, en 2004, le gouvernement Berlusconi a introduit une nouvelle réforme qui prévoit l'augmentation de l'âge minimum de départ à la retraite : à 60 ans à partir de 2008 et à 61 à partir de 2010. De plus, en 2015, le gouvernement devra décider si augmenter ultérieurement l'âge de départ à 63 ans.

L'analyse des impacts de la réforme Berlusconi est effectuée dans le deuxième papier. Nous montrons que les impacts sur la situation financière du système de retraite sont très positifs à court terme et à moyen terme, alors que, à partir de 2040, cette reforme devient complètement inefficace. Etant donné que l'augmentation de l'âge de la retraite prévue par le gouvernement Berlusconi ne sera pas suffisante à rétablir l'équilibre du système de retraite à long terme, nous avons considéré d'autres réformes, complémentaires à la réforme Belusconi, qui pourraient résoudre problème d'équilibre de long terme. Nous avons d'abord considéré des politiques migratoires. Nous avons montré qu'une politique migratoire ayant comme but d'atteindre l'équilibre du système de retraite à long terme n'est pas faisable du point de vue politique, car elle impliquerait la quadruplication du nombre annuel d'immigrés. Nous avons aussi montré qu'une politique migratoire sélective ne permet pas de résoudre le problème de long terme.

Dans le but d'atteindre l'équilibre du système de retraite à long terme, nous avons considéré une politique qui prévoit la réduction de la valeur de la pension. Nous avons montré que cet objectif serait atteint par une réduction de 11% du ratio de remplacement.

Dans la deuxième partie de la thèse nous avons analysé une méthodologie d'intégration entre les modèles macro d'équilibre général et les modèles de microsimulation. Les modèles d'équilibre général se basent sur un agent représentatif qui implicitement, mais sans aucun support théorique, devrait agréger le comportement des individus. Les modèles de microsimulation, par contre, permettent de prendre complètement en considération l'hétérogénéité présente dans les micro-données. Cependant, les modèles de microsimulation négligent les effets d'équilibre général, c'est-à-dire que l'analyse des impacts d'une politique sur le comportement individuel est effectuée sans considérer les effets de retour sur le prix d'équilibre. L'intégration entre les modèles macro d'équilibre général et les modèles de microsimulation, qui permet d'éviter les points faibles de deux méthodologies, apparaît donc une étape fondamentale dans l'évaluation des impacts d'une politique et en particulier des effets redistributifs.

Dans cette thèse, le lien «micro-macro» est obtenu en utilisant le résultat théorique d'agrégation présenté par Anderson, de Palma et Thisse (1992). Cette théorie d'agrégation montre que les individus, qui doivent choisir une quantité continue parmi un ensemble d'alternatives discrètes, peuvent être agrégés en un individu représentatif ayant des préférences CES.

Dans le modèle macro d'équilibre général, il est donc suffisant d'introduire un individu représentatif dont le comportement agrège parfaitement le comportement des individus. L'équilibre obtenu dans le modèle d'équilibre général, grâce aux propriétés d'agrégation, sera donc correct et pourra être injecté dans le modèle de microsimulation pour déterminer les changements du comportement et du revenu de chaque individu et, en particulier, pour analyser l'impact sur la distribution des revenus et sur les inégalités.

Dans le troisième papier nous avons montré l'utilité de cette méthodologie dans un contexte de vieillissement de la population d'une économie fictive. Nous avons utilisé un simple modèle OLG et une base de données micro générée par ordinateur afin d'évaluer les impacts potentiels du vieillissement de la population sur l'évolution de la distribution des revenus et des inégalités. Dans cette économie fictive, les individus doivent choisir entre travail et loisir et, dans le cas où ils décident de travailler, ils doivent choisir dans quelle profession. Nous avons regroupé les individus sur la base des caractéristiques démographique (âge, distingué en 5 classes, et sexe). Pour chaque groupe, nous avons déterminé une fonction d'offre de travail relative à chaque profession qui agrège parfaitement les choix des individus et qui est introduite dans le modèle OLG.

Nous avons simulé dans le modèle OLG un choc démographique qui provoque une augmentation des salaires et une réduction du taux d'intérêt. Ensuite, les prix d'équilibre ont été injectés dans le modèle de microsimulation, ce qui nous a permit de déterminer le changement de comportement de chaque individu.

Après avoir montré que les propriétés d'agrégation sont respectées, nous avons effectué une analyse sur la distribution des revenus et sur les inégalités dans cette économie fictive.

Dans le quatrième papier, nous avons appliqué cette méthodologie à l'économie Canadienne dans le but d'évaluer les impacts du vieillissement démographique sur le système macroéconomique et, en particulier, sur la distribution des revenus. Nous avons utilisé la base de données FMGD (Fichier de micro-données à grande diffusion) de l'année 2001.

Nous avons estimé les choix individuels concernant (1) travail - loisir ; (2) la profession ; (3) le niveau d'éducation pour les individus appartenant à la première classe d'âge (15-24). En particulier, nous avons considéré 10 types de professions et 5 niveaux d'études. Les individus ont été regroupés sur la base des caractéristiques (âge, sexe et niveau d'éducation). Par conséquent, le modèle OLG comprend 50 agents représentatifs qui agrègent parfaitement le comportement individuel en terme d'offre de travail par profession et de choix de type de diplôme.

Ensuite, nous avons introduit dans le modèle OLG le choc démographique, i.e. l'évolution des taux de fécondité et de mortalité que nous avons calibré dans le but de reproduire l'évolution démographique Canadienne. Les résultats du modèle OLG, concernant l'évolution des salaires d'équilibre pour chaque profession et celle du taux d'intérêt, ont été injectés dans le modèle de microsimulation afin d'analyser l'impact du vieillissement sur la distribution des revenus et le niveau d'inégalité mesuré par les indices de Gini.

L'approche «micro-macro» utilisée dans la deuxième part de la thèse fournit un double apport à la littérature existante : les paramètres des fonctions qui décrivent le comportement de l'agent représentatif dans le modèle d'équilibre général ne sont pas calibrés mais estimés sur des micro-données. Deuxièmement, cette approche permet d'effectuer une analyse au niveau individuel des effets d'une politique sur comportement de chaque individu et, par conséquent, sur la distribution des revenus.